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Comparison of Safety Factors Obtained from Limit Equilibrium Methods and Finite Element Analyses

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ABSTRACT: Designing with limit equilibrium methods involves a factor of safety as a measure of stability, i.e. how far the resisting structure is from the limit state, on the safe side. On the other hand, current engineering practice employs finite element programs (such as PLAXIS) which, instead of using a conventionally-defined factor of safety, reduces the overall shear strength of the soil by introducing a reduction factor that is applied to soil strength (i.e. tan ϕ and c values). Such fundamental differences in the incorporation of safety into the design process leads to different results from limit equilibrium methods and finite element simulations. In this parametric study, retaining walls, both cantilever and supported at a single level, with variable height and soil properties, are analyzed as example problems. For each case, multiple factors of safety are determined using different limit equilibrium methods found in the literature. In computer models, safety factor is defined in a variety of ways, based on; working and ultimate loads or stresses, normal & shear stresses along failure planes and reduction of soil strength.The safety factors from different methods are compared and relations between them were investigated by least squares regression analysis. Consequently, correlations among and between the safety factors obtained from limit equilibrium methods and finite element modeling results are proposed. Results of different limit equilibrium solution procedures are found to be linearly correlated with each other without involving soil strength parameters. Success of the FEM-based methods is found to vary based on the soil and support type.

1. INTRODUCTION

In modern engineering, traditional solution procedures based on analytical or empirical methods are no longer able to handle complex problems. Such level of complexity can only be matched by using finite element softwares, which provide rapid, accurate and feasible solutions compared to conventional solution procedures. On the other hand, as the softwares get more complex and closer to the real situation, insight of most daily users towards them decreases, which brings the need of associating results derived from conventional solutions and software outputs.

One of the most common geotechnical fields that involve such problems are retaining walls. Traditional analysis methods for these structures are presented in various codes. Also, these structures can be modelled with recent softwares using numerical modeling.

An embedded retaining wall is a relatively thin geotechnical structure that is made of piles that are inserted into (e.g. steel sheet piles) or constructed in (e.g. reinforced concrete bored piles) the ground and designed to withstand imbalance of lateral forces due to dredging, backfilling, pore pressures and surcharges. The wall may withstand these forces either by passive earth pressures only (cantilever retaining walls) or in addition to passive earth pressures, structural supports such as anchors and struts may be used (anchored – propped retaining walls).

Traditional approaches involve application of an overall safety factor to the calculations in order to account for the effects of all the unknowns (such as uncertainties in soil properties and loads, construction tolerances, unplanned excavation concept in Eurocode 7 (European Norm BS EN 1997-1:2004), ultimate limit state design, differences between modelled and actual conditions of pressures etc.) in the design. FS can be applied using; a scale factor applied to the depth of embedment, calculated from limit equilibrium methods; a reduction factor applied to the theoretical soil strengths; or a factor applied to net or gross pressures.

PLAXIS is one of the most common geotechnical modelling softwares used in today's practice. It is a non-linear finite element computation software used for two or three dimensional analysis of deformation and stability in geotechnical engineering.

In this paper, factor of safety (FS) values are calculated and analysed for an embedded retaining wall from Limit Equilibrium (LE) Methods and 2D Finite Element Modeling. Various assumptions of lateral earth pressure distribution are considered for LE solutions. In PLAXIS calculations, FS is calculated using; (i) overall strength reduction factors, (ii) ratio of passive forces obtained from ultimate and working state stresses that are accumulated in excavated side of the wall, (iii) normal and shear stresses over the most likely failure surfaces defined according to displacements and (iv) gradual reduction of overall soil strength of only the excavated side. These four approaches to define FS using PLAXIS simulations have originally been used by Yaman G. et. al. (2009) for the design of the retaining system for the foundation excavation of 69 Tower in Tripoli, Libya.

2. METHODOLOGY FOR LIMIT EQUILIBRIUM SOLUTIONS

Pile walls are flexible structures that allow displacement in soil; which results in reaching limiting values of earth pressures. These structures satisfy equilibrium conditions with passive pressures and/or with help of one or more rows of struts or anchors that provide some fixity. Unless there is a structural failure, the expected failure mechanism of an embedded cantilever retaining wall is as shown in figure 1a. For a cantilever wall with a depth of penetration d, rotation at a point z_p below the excavation level is expected. Limiting stress distribution consistent with this mode of collapse is illustrated in figure 1b (Bolton & Powrie, 1987). The unknowns d and z_p are determinable from horizontal force and moment equilibrium. It is assumed that the materials are plastic and the lateral earth pressures in each zone of soil at failure are given by the active and passive limits.

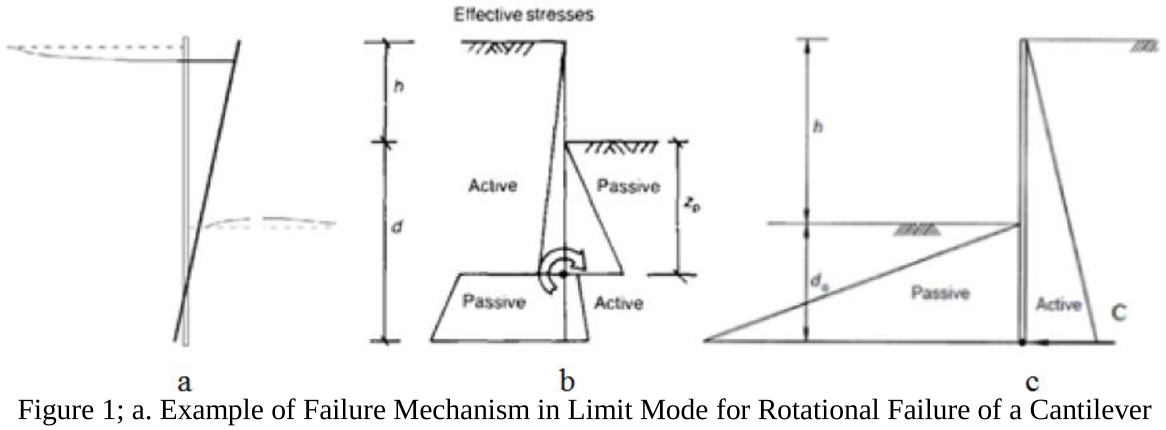


Figure 1; a. Example of Failure Mechanism in Limit Mode for Rotational Failure of a Cantilever Embedded Retaining Wall (Sigström and Persson, 2010), b. Equilibrium Stress Distribution of a Cantilever Retaining Wall at Limit Equilibrium (Bolton & Powrie, 1987), c. Simplified Pressure Distribution of a Cantilever Wall (California Dept. of Transportation, 1995)

In addition to the traditional pressure diagram shown in figure 1b, a "simplified" pressure distribution can be used in calculations. In this pressure distribution (figure 1c), rotational point is assumed to be at 5/6 of embedment depth. Stresses below the level of rotational point are omitted and a horizontal force is placed at bottom of the pile in order to replace the omitted forces, and their moment imbalance is ignored as their moment arms are small.

Different types of LE solution procedures according to their pressure distribution diagrams can be used in calculations. For cantilever models, solutions are obtained for lateral earth pressure diagrams with traditional (LE 1) and simplified (LE 2) distributions. For one level strut-supported models, free earth support method (wall toe is free to move) is used. Limit equilibrium procedures followed in the calculations here are also presented in various manuals and table 1 presents the list of manuals with corresponding LE solution in this paper.

Table 1. Manuals involved in LE calculations with corresponding LE solution

	Models of Cantilever Walls				Models of One Level Strut Supported Walls	
	Retaining Sand		Retaining Clay		Retaining Sand	Retaining Clay
	Traditional	Simplified	Traditional	Simplified	Free Earth Support Method	
California Trenching and Shoring Manual (1995)	LE 1	LE 2	LE 1	LE 2	-	-
U.S. Army Corps of Engineers (1994)	-	-	-	-	LE	-
U.S. Dept. Of Transportation (1984)	-	-	LE 1	-	-	LE

3. METHODOLOGY FOR FINITE ELEMENT MODELING COMPUTATIONS

PLAXIS software package includes "phi-c reduction" as a tool to measure the safety of analyzed problems. In this mode, the program repeats the analysis while dividing the values of c and tan ϕ by a gradually increasing factor. In this manner, phi-c reduction method provides an overall safety factor for the model. Results of this analysis is referred in this paper as "PLX 1".

Phi-c reduction analysis results are not straightforwardly obtained. An important point to be considered in interpreting the results of phi-c reduction is whether the number of analysis cycles is sufficient to converge to a single result. Also, while working with undrained clay models, after the analysis converges to an FS, after a number of cycles, the value drops excessively. Outputs must be thoroughly examined in order to avoid misleading results.

In addition to phi-c reduction method, 3 methods are developed for obtaining safety factors.The second methodology (PLX 2) provides FS values from the ratio of Ultimate Passive Forces to Working State Passive Forces. Lateral earth pressures acting on the wall are different in limit equilibrium analyses and PLAXIS outputs. Limit equilibrium analyses consider the ultimate case with fully active and passive lateral pressures. On the other hand, PLAXIS calculates the stresses at the working state.

The passive pressures calculated by PLAXIS are the stress applied to the soil by the wall, and the passive pressures given by Rankine distribution are the available resistance. Ratio of resultants of these passive stresses presents the FS value obtained from this method of analysis. This is akin to the FS definition from limit equilibrium analysis given in many sources (i.e. applying FS to passive forces). Therefore, for PLX 2, FS is defined as;

$$FS_{PLX2} = \frac{\sum F_{\text{passive, limit equilibrium analysis}}}{\sum F_{\text{passive, plaxis outputs}}} \qquad (1)$$

In the third analysis method (PLX 3), shearing planes along which soil is likely to fail are examined. Stresses along these planes are calculated and the safety factor of the model is derived as the ratio of available shear strength along the planes and working state shear stresses, similar to sliding wedge analysis for slopes.

Shearing planes are determined from the displacement outputs at the end of phi-c reduction analysis, which provides visualisation of these failure planes. In determining shearing planes of cohesionless models, the failure plane is not clear, even after phi-c reduction analysis. Therefore, shearing planes are selected as $45^\circ + \phi/2$ and $45^\circ - \phi/2$ from the horizontal, as estimated by Rankine Theory (i.e. on active and on passive side, respectively).

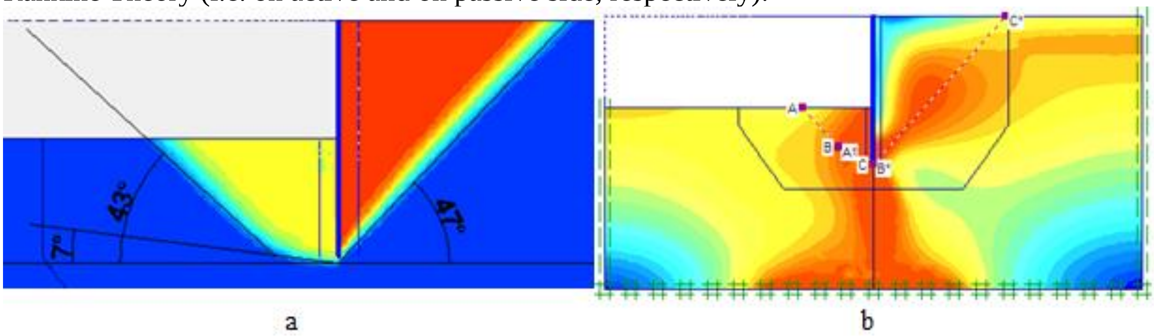


Figure 2; a. Total displacements of an undrained clay model failed after phi - c reduction, b.Shearing planes applied onto the output

After determining the planes by using displacement outputs as shown in figure 2a, the coordinates of these planes are defined in PLX and stresses along them are obtained. Figure 2b presents the potential shear failure planes, drawn at the end of working state analysis, from the results of this analysis, forces are calculated for each element along the shearing planes, by numerical integration, as;

$$FS = \frac{\int_0^D (c + \sigma \tan\phi)\, dL}{\int_0^D \tau \cdot dL} = \frac{c \cdot \int_0^D dL + \tan\phi \cdot \int_0^D \sigma \cdot dL}{\int_0^D \tau \cdot dL} \tag{2}$$

Where, c is the cohesion, ϕ is the internal friction angle of soil, dL is the length between two nodes along the shear plane where stresses are provided, σ and τ are the normal and shear stresses, respectively. In the fourth analysis method (PLX 4), solutions are obtained from the ratio of initial and reduced strength in excavated side of the model. Phi-c reduction method reduces the soil parameters step by step in the entire model, and determines the lowest strength parameter with which the section can withstand lateral earth pressures. However, the conventional hand solutions generally apply an FS only to the passive side. With the aim of catching a better correlation, instead of reducing the parameters on both sides of the wall, in this methodology, reducing the soil parameters of only the excavated side is investigated. For cohesionless models ϕ, and for undrained cohesive models c_u of soil in the excavated side is reduced gradually, in an iterative scheme, until ultimate limit state is reached. The FS is defined as;

$$FS_{cohesionless} = \frac{\tan\phi_{initial}}{\tan\phi_{reduced}} \qquad \& \qquad FS_{undrained} = \frac{c_{u,\ initial}}{c_{u,\ reduced}} \tag{3}$$

4. ANALYSIS RESULTS OF CANTILEVER WALLS RETAINING SAND

LE1 analysis always results in a higher FS as LE2 is a simplification on the safe side. Also, according to the calculations, depth of rotational point is almost independent of retained height H and internal friction angle ϕ, and it changes with embedment depth D. In sand, as the height of the cantilever retaining wall is increased, the relatively high displacements cause wedges to form in retained side of wall. This situation leads PLAXIS calculation module to mistake movements of these wedges as failure. In order to avoid this situation, instead of the default settings of the software, error tolerances for the iterative procedure must be manually increased. Wedges can be detected in the output module, as they are surrounded by Mohr-Coulomb points (plastic points) where $\tau = \sigma \cdot \tan\phi + c$.

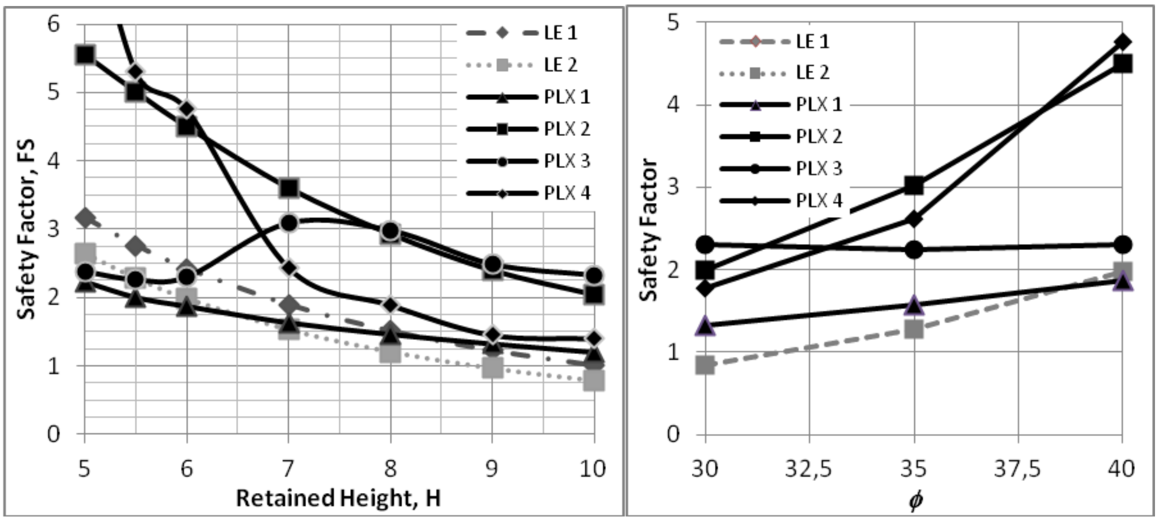


Figure 3 & 4; Variation of; FS with H for $\phi = 40°$, friction angle, ϕ with FS for H = 6m

As can be seen in figure 3, for small wall heights, results obtained from PLX 1 analyses are slightly lower compared to limit equilibrium methods. With increasing height, FS values from LE methods drop faster than PLX 1 results. Passive pressures are generated gradually with increasing retained heights, which provides PLX 1 FS results to be higher than limit equilibrium results for increasing heights.

PLX 2 results provide higher FS values compared to limit equilibrium and PLX 1 results. However, general trend of PLX2 results match well with the rest of the data set.

PLX 3 analyses on the other hand, provide unreliable results for sand. For heights of 6 and 7m, although the retaining height increases, FS seems to increase as well, which is an unrealistic situation. The reason for this is that the true failure planes might not match with the assumed $45° \pm \phi/2$ Rankine failure planes. Very little slope variations or non-linearities in shear plane formations result in great differences between real and calculated FS values. The difference of results between PLX 3 and other methods can also be seen in figure 4.

PLX 4 analyses provided very high results for safer conditions where retained height is relatively low. On the other hand, with increasing H, results converge to limit equilibrium results, as can be seen in figure 3. Also, for increasing ϕ values (safer conditions), PLX 4 results increase at a greater rate compared to the other methods (see figure 4).

5. ANALYSIS RESULTS OF CANTILEVER WALLS RETAINING CLAY

PLX 1 method provided consistently higher results compared to limit equilibrium methods throughout the cases of cantilever walls in clay, as opposed to sand models where PLX 1 results are below LE results for safer models. Results are in parallel with the LE results and follow the expected trend, as presented in figure 5.

Results obtained from PLX 2 analyses provided the lowest FS values, compared to other methods. This situation is because, instead of rotating around a fixed point as assumed in limit equilibrium methods, according to the FEM analysis, the wall translates horizontally into the excavated zone as well as rotating. In this situation, soil in the excavated area below dredge level provides higher passive forces compared to limit equilibrium methods. Since PLX 2 uses the ratio of passive forces obtained from limit equilibrium methods and PLX analysis results, FS values obtained are relatively low compared to other methods.

PLX 3 results provide higher FS values within the considered data set (see figure 5). Unlike the analyses in sand, shearing planes are visible through PLAXIS phi-c reduction outputs, (as in figure 2a) and it is easy to obtain the stresses using these planes with working stress analysis outputs.

Results for heights 7 and 8m are slightly off the general trend. Although the shearing planes are easy to locate, the lines to be used in deriving the most critical stresses are placed manually in the software, which may lead to small deviations from the main trend of results. PLX 3 results are in well accordance with PLX 1 results especially when models with FS close to 1 are considered.

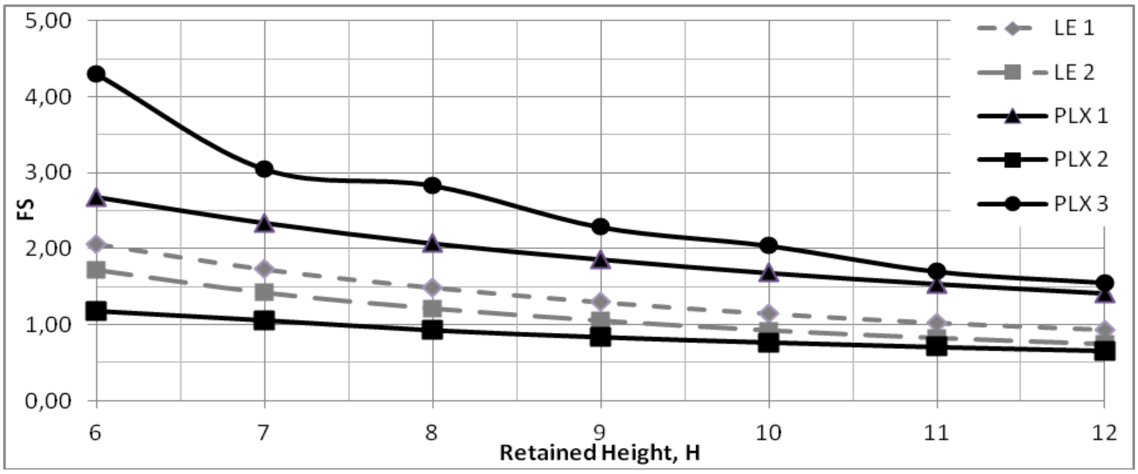
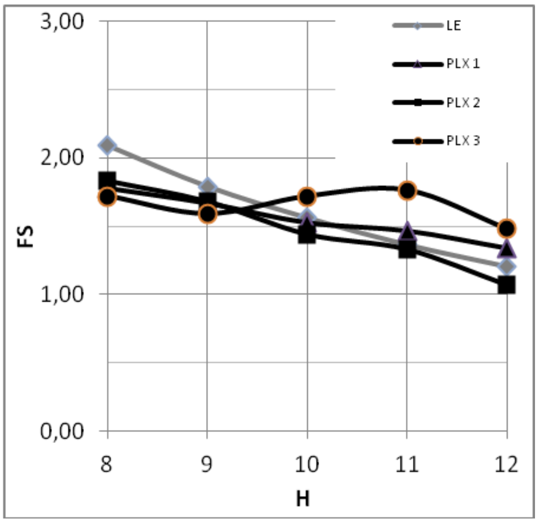


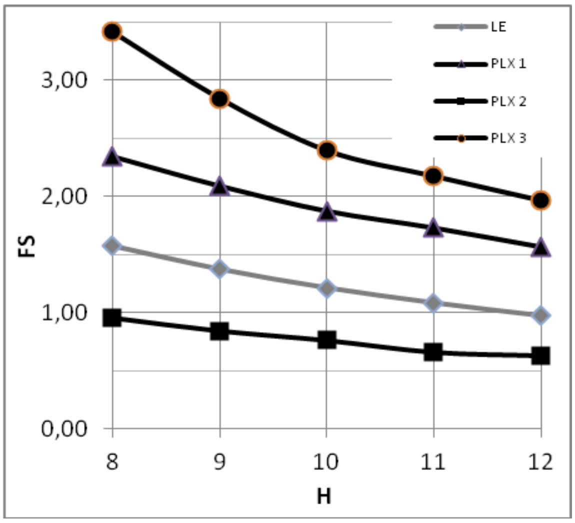
Figure 5; Variation of FS with H for c=75kPa

PLX 4 results provide the highest FS values of the data set (they are outside the scale of figure 5). Method involves the ratio of undrained strength in retained side and undrained strength reduced in excavated side where model is at the edge of collapse. This consideration leads to very high FS values for lower retaining heights where crack height is close to dredge level and little or zero active lateral pressure is generated. The excavation approaches being able to stay stable with an unretained vertical cut, FS value from this method approaches infinity.

6. RESULTS OF ONE LEVEL STRUT SUPPORTED WALL ANALYSES



a



b

Figure 6; Strut supported walls in; a. sand ($\phi = 35°$), b. clay (c=75kPa)

For single-level supported wall models in sand, plot of H vs FS for $\phi = 35^\circ$ is in figure 6. PLX 1 outputs provide similar FS values with limit equilibrium FS values. For less critical cases, PLX 1 provides lower FS values, hence, FS line representing PLX 1 outputs is below the limit equilibrium line (similar to cantilever sand results). For more critical cases where retained height is larger or internal friction angle is lower, PLX 1 results are above the limit equilibrium values. PLX 2 analyses provide very close results with limit equilibrium's, compared to PLX 1. Methodology developed in PLX 2 seems to work very well for strut supported walls retaining sand. PLX 3 method, on the other hand, provided fluctuating results. The reason is due to determination of shearing planes. Overall, results obtained from methods mentioned above are very close and forming similar lines, providing close results with small differences with each other, except for PLX 3.

Results obtained for strut supported clay models (figure 6b) show similarity in general trend, but the differences between the resulting FS are very high, compared to strut supported sand results. Limit equilibrium FS results are very close to unity. PLX 1 FS values are higher than LE's and providing the second highest result data set.

PLX 2 outputs provided very low results. Results of PLX 2 is almost parallel to those of LE's but it is below the limit equilibrium method's line of FS vs H. The results of PLX 2 are even below FS = 1 line where as LE results are higher.

PLX 3 results are providing the highest FS factors for models. Unlike the results of sand cases, PLX 3 results are reasonable and following the general trend of other solutions.

7. REGRESSION ANALYSIS

Linear regression involves direct relation of two different analysis results. An example for this type of results is provided in figure 7 for methods PLX 2 & PLX 3 of cantilever wall retaining undrained clay. Corelations derived from the FS results are listed in table 2.

In comparison of analysis pairs where linear regression is not sufficient to obtain correlations with high R^2 values, different correlations maybe obtained by incorporating soil strength parameters into calculations.

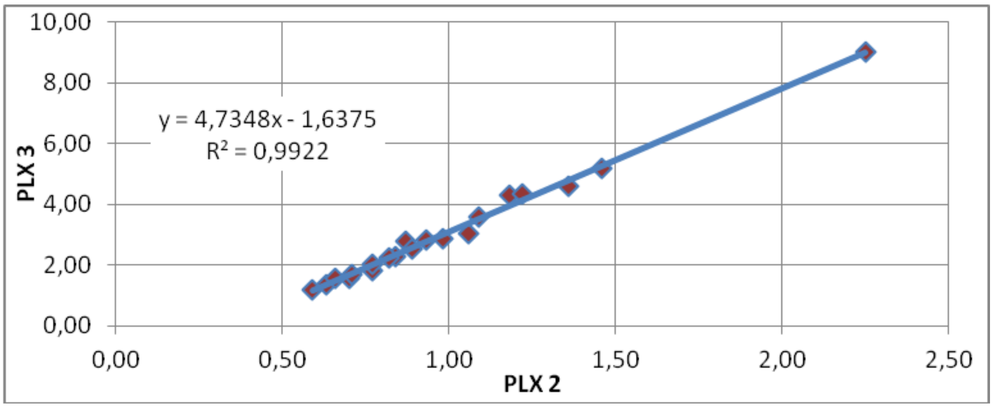


Figure 7; Cantilever wall retaining undrained clay, FS $_{PLX2}$ vs FS $_{PLX3}$, for all c_u values

8. CONCLUSIONS

In this parametric study, factors of safety (FS) for simple retaining walls are calculated by multiple limit equilibrum (LE) methods and four different finite element-based calculation procedures (PLX).

Limit equilibrium methods provide very close FS values among themselves. The major difference to be considered within limit equilibrium analyses is due to simplification of earth pressure distributions. Also, for the investigated data set, depth of rotational point is found to be almost independent of retained height H and internal friction angle ϕ, and it changes with embedment depth D.

Table 2. Linear corelations obtained from results of FS calculations

Cantilever Wall in Sand

$FS_{PLX2} = 1.6326*FS_{LE1} + 0.4055$

$FS_{PLX2} = 1.686*FS_{LE2} + 0.4146$

Cantilever Wall in Clay

$FS_{PLX1} = 4.7348*FS_{PLX2} - 1.6375$

$FS_{PLX3} = 2.66*FS_{LE1} - 0.4912$

$FS_{PLX3} = 2.2733*FS_{LE2} - 0.6507$

Strut Supported Wall in Sand

$FS_{PLX1} = 0.3872*FS_{FES} + 0.9107$

$FS_{PLX2} = 0.8232*FS_{FES} + 0.1372$

$FS_{PLX2} = 2.0818*FS_{PLX1} - 1.729$

$FS_{PLX3} = 0.1982*FS_{FES} + 1.2935$

$FS_{PLX3} = 0.5145*FS_{PLX1} + 0.8232$

$FS_{PLX3} = 0.2355*FS_{PLX2} + 1.2687$

Strut Supported Wall in Clay

$FS_{PLX2} = 0.5218*FS_{FES} + 0.1296$

$FS_{PLX2} = 0.3504*FS_{PLX1} + 0.1025$

In predicting FS values in sand, for models with relatively safer parameters (lower retaining heights and higher soil strength parameters), PLX1 FS values are slightly lower than those of LE calculations, due to forming of passive pressures. For relatively critical models where FS values approach to 1, PLX1 provides higher FS values than LE methods. For clay models, FS values from PLX1 are consistently higher than LE results.

Results of PLX2 provided higher FS values for cantilever sand models compared to those of PLX1 and LE, whereas it gives lower FS values in the case of clay.

PLX3 method gives results that are not in accordance with results of other methods for sand models. In clay models, PLX3 results fit well with the rest.

PLX4 results can be related to other FS values for sand cases and results are close to those of LE and PLX1, especially in critical cases where FS approaches 1. On the other hand, for clay cases where undrained shear strength is high and retained height is low compared to other models, PLX4 results provide very high FS values.

In regression analyses, safety factors obtained from each method in all the computations mentioned above are plotted with respect to each other. These results are used in a least squares linear regression analysis relating each pair of safety factor to each other whenever possible.

Results showed that, for cantilever sand models, among limit equilibrium FS data sets, it is possible to obtain linear relations. Data pairs involving limit equilibrium and PLX3 outputs fit linear equations which are independent of soil properties (except for strut supported sand models). This shows that, shearing planes that are selected from PLAXIS outputs for calculating PLX3 FS values are close to real situation, and hence these FS values are in well accordance with limit equilibrium's FS values.

Strut supported sand model's data pair relations all form linear fits. For these models, R^2 values of correlations between limit equilibrium solution and PLAXIS solutions are high. The only exception is PLX3, since obtained data have incompatibilities due to assigned shearing planes.

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