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# Comparison of formulae for shear strength of unsaturated soils

Kaveh Hassan Zehtab
*PhD. Candidate, Middle East Technical University, Ankara, Turkey, kaveh.zehtab@metu.edu.tr*

Nabi Kartal Toker
*Instructor Dr., Middle East Technical University, Ankara, Turkey, toker@metu.edu.tr*

Mohammad Ahmadi Adli
*PhD. Candidate, Middle East Technical University, Ankara, Turkey, e174832@metu.edu.tr*

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ABSTRACT: During last 60 years many research studies have been conducted on calculation of shear strength of unsaturated soils and effects of matric suction on it. Several formulae are developed for predicting shear strength of unsaturated soils, based on strength tests on unsaturated soils. A variety of experimental techniques and laboratory equipment are used for unsaturated shear strength characterization. This study discusses the accuracy of shear strength prediction for unsaturated soils with three different formulae. The predicted shear strengths of three different soils for a wide range of suction, using these formulae are compared. And finally the dependency of accuracy of the best formula among the three investigated formulae on the net normal stress is investigated.

## 1 INTRODUCTION

In any engineering problem, where stability of a given soil is concerned (e.g. bearing capacity, slope stability, lateral earth pressure, pavement design, and foundation design), defining the shear strength of soil is crucial. Therefore, during last 60 years many research studies has been conducted on shear strength of unsaturated soils and effects of matric suction on it. Two approaches that are generally used for studying shear strength of unsaturated soils are: Effective Stress Approach, which nucleated from Bishop (1959)'s extension of Terzaghi's principle of effective stress to unsaturated soils, and Independent Stress State Variables Approach, which was formulated for the first time by Fredlund and Morgenstern (1977) to consider matric suction and total normal stress separately.

In the effective stress approach, shear strength is based on modifying the effective stress term in the Mohr-Coulomb failure criterion, as shown below (Bishop et al., 1960):

$$\tau = c' + [(\sigma_n - u_a) + \chi.(u_a - u_w)].\tan\phi' \qquad (1)$$

where: $\tau$ is shear strength of unsaturated soil, $c'$ is effective cohesion, $\phi'$ is angle of frictional resistance, $(\sigma_n\text{-}u_a)$ is net normal stress, $(u_a\text{-}u_w)$ is matric suction and $\chi$ is a soil parameter with values between zero and unity depending on the degree of saturation. $\sigma_n$ in equation 1 is total stress, $u_a$ is pore-air and $u_w$ is pore-water pressure. Several studies have been conducted theoretically (e.g. Atchison, 1960) and experimentally (e.g. Donald, 1960) in order to formulate $\chi$ with respect to different parameters related to degree of saturation since then.

On the other hand, independent stress state variables approach formulates shear strength of unsaturated soil as a function of net normal stress $(\sigma_n\text{-}u_a)$, and matric suction $(u_a\text{-}u_w)$ as shown in equation 2.

$$\tau = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \qquad (2)$$

where $\phi^b$ is the shear strength contribution due to matric suction, and it changes nonlinearly with matric suction (Gan et al., 1988; Escario and Juca, 1989).

Since unsaturated soil tests are very time consuming and expensive, different researchers attempted to formulate shear strength of unsaturated soils using soil-water characteristic curves and saturated strength parameters ($c'$ and $\phi'$). Soil-water characteristic curve can be represented in terms of volumetric water content ($\theta_w$), saturation degree ($S$), or gravimetric water content ($w$). This curve provides the interpretative tool for understanding the behavior of unsaturated soil. All of the procedures for predicting shear strength of unsaturated soils used in this study use the soil-water characteristic curves.

Three procedures (Fredlund et al., 1995; Vanapalli et al., 1996; Öberg and Sallfors, 1995) for predicting shear strength of unsaturated soils are selected to be compared in this study. The predicted shear strength values will be compared by results of extensive direct shear tests on three different soils by Escario and Juca (1989). The tested soils have different gradation and plasticity index and are tested under different net normal stresses and matric suctions. A similar comparison was done by Vanapalli and Fredlund (2000); however, in that study results of tests with only one net normal stress was taken into account. In this paper the accuracy of the three formulae is compared and suggestions are made in order to increase precision of the initially most accurate formula (Fredlund et al., 1995).

## 2 EXPERIMENTAL DATA

In an extensive study conducted by Escario and Juca (1989), shear strength of three different soils was measured under different net normal stresses. Shear strength is measured using a direct shear device under controlled suction. The direct shear is placed inside a pressure chamber to which pressurized nitrogen can be introduced via top of specimen through a coarse grained porous stone. The bottom side of the specimen is in contact with water at atmospheric pressure through a high air entry value porous stone, or a semipermeable membrane for higher values of suction. When the equilibrium is reached the suction in pore-water is equal to the applied air pressure. Loads and displacements are monitored via appropriate push rods. Two of the tested soils, namely, Madrid gray clay and Guadalix red clay (Red clay) were tested under suctions ranging between 0 to 15,000 kPa and the other soil, Madrid clayey sand was tested under suctions ranging between 0 to 5,000 kPa. Properties of three soils are shown in table 1. Also the void ratio *(e)* and specific gravity *($G_s$)* of these soils are reported by Vanapalli and Fredlund (2000) which are included in table 1.

Table 1. Properties of the soils tested by Escario and Juca (1989).

| Soil name | # Pass (%) | | | | LL(%) | PI(%) | e | $G_s$ |
|---|---|---|---|---|---|---|---|---|
| | 10 | 16 | 40 | 200 | | | | |
| Clayey sand | 100 | 94 | 48 | 17 | 32 | 15 | 0.38 | 2.71 |
| Gray clay | - | - | 100 | 99 | 71 | 35 | 1.03 | 2.64 |
| Red clay | - | 100 | 97 | 86.5 | 33 | 13.6 | 0.48 | 2.66 |

Escario and Juca (1989) have measured and presented the soil-water characteristic curves of three soils up to 15,000 kPa using calibrated springs with an applied stress of 20 kPa that provides the necessary contact between specimen and ceramic stone or pressure membrane. Nevertheless, to achieve the entire soil-water characteristic curve data as suggested by Vanapalli and Fredlund (2000) curves are fitted on the measured data of soil-water characteristic curve using equation 3 (Fredlund and Xing, 1994) and extended to 1,000,000 kPa and is shown in figure 1.

$$\theta_w(\psi) = \theta_s \left( 1 - \frac{\ln\left(1 + \frac{\psi}{h_r}\right)}{\ln\left(1 + \frac{10^6}{h_r}\right)} \right) \left( \frac{1}{\ln\left(\exp(1) + \left(\frac{\psi}{a}\right)^n\right)^m} \right) \quad (3)$$

where: $\psi$ is soil suction, $\theta_w$ is volumetric water content, $\theta_s$ is saturated volumetric water content, $a$ is suction related to the inflection point on the curve, $n$ is soil parameter related to slope at the inflection point, $m$ is soil parameter related to the residual water content and $h_r$ is soil suction in kPa related to the volumetric residual water content ($\theta_r$). Parameters $a$, $n$ and $m$ are to be obtained by fitting the equation on experimental data.

A graphical procedure is followed in order to define residual state saturation degree and suction as described in Vanapalli and Fredlund (2000), figure 1. The residual saturation can be defined as the point where the tangent line to soil-water characteristic curve extending from 1,000,000 kPa intersects the tangent line to it at the transition zone (just after air entry value) where the soil tends to behave linearly, (Vanapalli and Fredlund, 2000). A computational technique can also be used to determine the residual suction value.

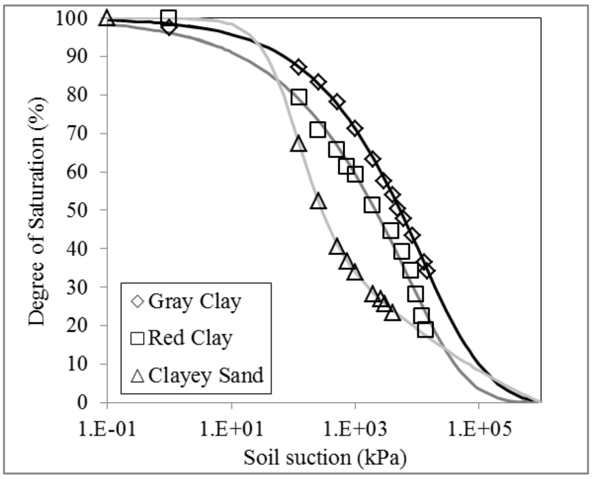


Figure 1. Soil-water characteristic curves for three soils tested by Escasrio and Juca (1989).

The residual suctions and saturation degrees for three soil type along with saturated shear strength parameters are all shown in table 2.

Table 2. Soil-water characteristic curve data and saturated strength parameters of the experimental data (Vanapalli and Fredlund, 2000).

| Parameter | Gray clay | Red clay | Clayey sand |
|---|---|---|---|
| Residual $\psi$, kPa | 40000 | 33000 | 12000 |
| Residual $S$ (%) | 23 | 16 | 17.6 |
| $c'$ (kPa) | 30 | 20 | 40 |
| $\phi'$(degrees) | 25.3 | 34 | 39.5 |

## 3 FORMULAE FOR PREDICTING SHEAR STRENGTH OF UNSATURATED SOILS

Dealing with the nonlinear variation of the parameter $\phi^b$ with respect to suction in independent stress state variables approach was a challenging subject for so many researchers during past years. New strength models have been developed in order to take this nonlinearity in to account in a practical manner. One such model is suggested by Fredlund et al. (1995), as equation 4. This nonlinear formula is developed to predict shear strength of unsaturated soil using the entire soil-water characteristic curve and is claimed to be a general approach fitting with several soil types.

$$\tau = \left(c' + \left(\sigma_n - u_a\right)\tan\phi'\right) + \left(\left(u_a - u_w\right)\left\{\Theta^{\kappa} \cdot \tan\phi'\right\}\right) \quad (4)$$

where: $\Theta$ is normalized water content ($\theta_w/\theta_s$) and $\kappa$ is fitting parameter with which a best fit between the predicted and experimental data can be established. This formula transforms $tan\phi^b$ term in independent stress state variable approach in to the form of $\Theta^{\kappa}.tan\phi'$. This way the nonlinearity of shear strength contribution due to suction ($tan\phi^b$) in transition zone can be incorporated in the formula. However, using this formula, the condition in which $tan\phi^b$ up to air entry level is equal to $tan\phi'$ will be held as well. This formula is able to predict shear strength of unsaturated soil from a fully saturated condition to a total dry condition.

The second considered formula in this study is suggested by Vanapalli et al. (1996) where no fitting parameter is used. As it is shown in equation 5, the normalized water content is instead replaced with a more general term containing residual and saturated water contents or degrees of saturation of soils.

$$\tau = \left(c' + \left(\sigma_n - u_a\right)\tan\phi'\right) + \left(\left(u_a - u_w\right)\left\{\left(\frac{\theta_w - \theta_r}{\theta_s - \theta_r}\right)\tan\phi'\right\}\right) \quad (5)$$

where: $\theta_w$ is volumetric water content, $\theta_s$ is saturated volumetric water content and $\theta_r$ is residual volumetric water content. As explained before the value of $\theta_r$ has to be estimated from soil-water characteristic curve.

The third formula unlike equations 4 and 5 which are based on independent stress state variable approach satisfying continuum mechanics concept is using Bishop's effective stress concept. In the proposed formula by Öberg and Sallfors (1995, 1997) $\chi$ in Bishop's effective stress approach (equation 1) is replaced by degree of saturation ($S$), (equation 6).

$$\tau = c' + \left(\sigma_n - u_a\right)\tan\phi' + \left(u_a - u_w\right)\left(S \cdot \tan\phi'\right) \quad (6)$$

Since S is the part of pore area which is occupied by water it can approximate the value of $\chi$. This formula is primarily for prediction of shear strength of non-clayey soils.

Although there are numerous formulae for predicting shear strength of unsaturated soils only formulae shown in equations 4, 5 and 6 are used in this study due to resulting accuracy regardless of their simplicity. It should be noted that equation 4 due to having a fitting parameter ($\kappa$), in order to predict shear strength of unsaturated soil needs experimental results on unsaturated soil in addition to soil-water characteristic curves and saturated strength parameters.

## 4 COMPARISON OF PREDICTED AND EXPERIMENTAL SHEAR STRENGTHS

In this section predicted shear strength using equations 4, 5 and 6 are compared to those experimentally obtained by Escario and Juca (1989) and results are shown in figures 2, 3 and 4.

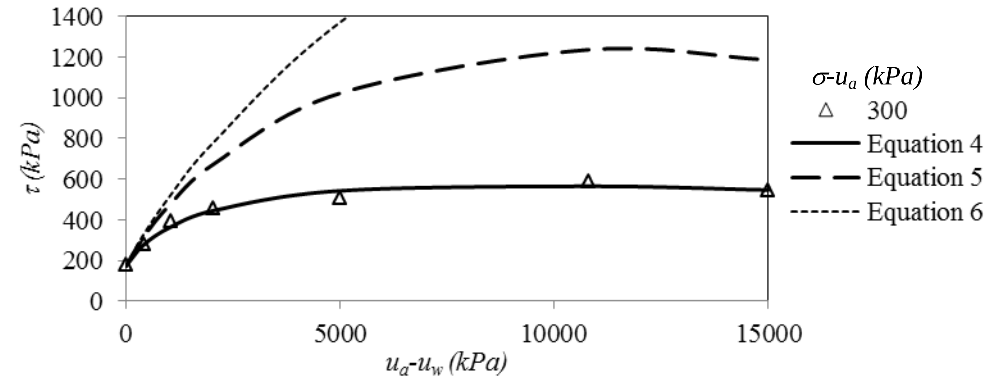


Figure 2. Comparison of predicted and experimental shear strength of Madrid gray clay, using different formulae for large suction range under a single net normal stress.

Figure 2 compares predicted and experimental shear strength of Madrid gray clay, using equation 4, 5 and 6 for a suction range between 0 and 15,000 kPa under single net normal stress equal to 300 kPa. It should be taken in to account, $\kappa$ in equation 4 is to be fitted on results of tests under all net normal stresses. However, since tests on gray clay are conducted under one single net normal stress, $\kappa$ is produced by fitting equation 4 on results of tests under the only net normal stress. As shown in figure 2, it models shear strength much more accurate than it generally does. Equation 5 in comparison with equation 6 predicts better the shear strength of this soil. Hence, both equations 5 and 6 give non-conservative shear strengths. It should be taken in to account that the experimental data provided by Escario and Juca (1989) are limited to only one net normal stress for this soil type which can be reason of high accuracy of equation 4.

Figure 3 compares predicted and experimental shear strength of Red clay, using equation 4, 5 and 6 for a suction range between 0 and 15,000 kPa under three different net normal stresses 120, 300 and 600 kPa. As shown in figure 3, equation 4 gives the best prediction of the shear strength of Red clay for almost all suction ranges and after that equation 5 yields to a better predictions than equation 6 similar to the case of Gray clay. It should be noticed that the ($\kappa$) in equation 4 for case of Red clay is not adjusted for experimental data of any one net normal stress; but instead, the entire data set is used for determining the best-fitting value. Therefore, the accuracy of equation 4 varies with net normal stresses. As it can be observed in figure 3, the shear strength is modeled better for net normal stress of 300 kPa compared to those of 120 and 600 kPa. This observation suggests influence of net normal stress in value of ($\kappa$).

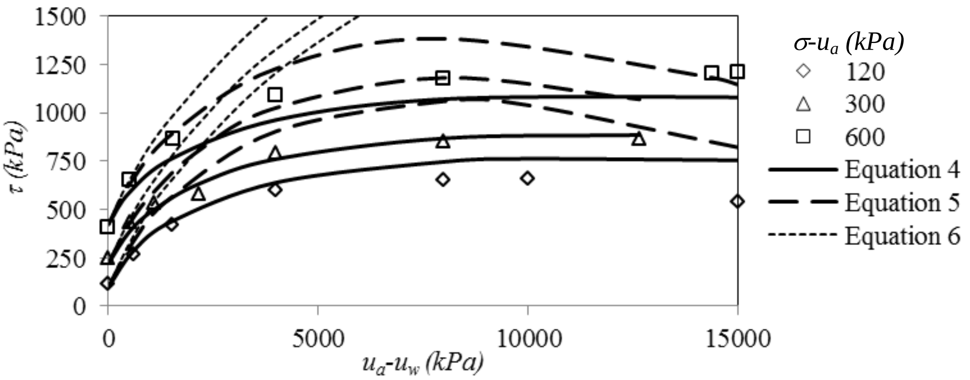


Figure 3. Comparison of predicted and experimental shear strength of Red clay, using different formulae for large suction range under three different net normal stresses.

Figure 4 compares predicted and experimental shear strength of Madrid clayey sand, using equation 4, 5 and 6 for a suction range between 0 and 4,000 kPa under two different net normal stresses, 120 and 600 kPa. In this case unlike for Gray and Red clays equation 5 predicts shear strength in a better manner than those of equation 4 and 6. For suctions higher than 1,000 kPa equation 4 predicts the shear strength better than equation 5. Below 1,000 kPa suction, the predicted shear strengths by equation 4 is more conservative than equation 5, which predicts almost the exact experimental values. However the predicted shear strength values by equation 4 are less than 10% lower than experimental ones. It should also be taken into account that the suction range for experimental data in case of Clayey sand is limited to 4,000 kPa.

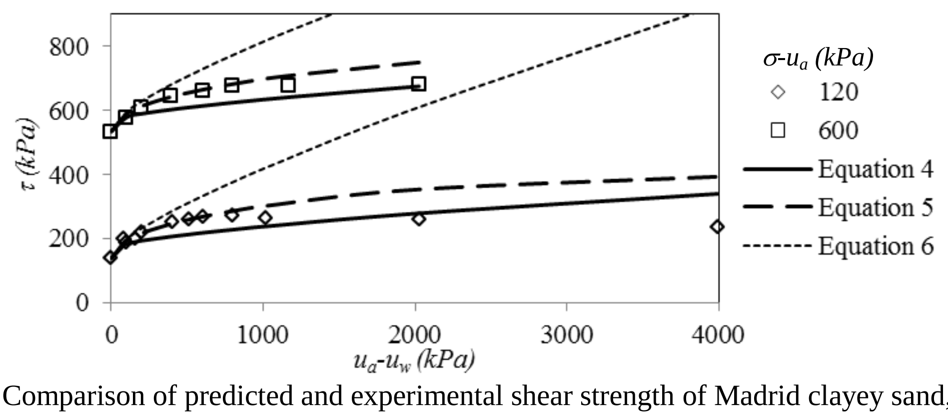


Figure 4. Comparison of predicted and experimental shear strength of Madrid clayey sand, using different formulae for large suction range under two different net normal stresses.

## 5 DISCUSSION OF RESULTS

Comparing the predicted shear strengths by three formulae (equations 4, 5 and 6) for three different soils with experimental data as presented in figures 2, 3 and 4 suggests equation 4 as the one that yields to the best results. This conclusion was expected since equation 4 uses a fitting parameter ($\kappa$) that adjusts the formula to experimental data which is not used by the other formulae. It is also observed for cases in which equation 4 is adjusted to results of experimental data under different net normal stress it predicts shear strength of those with normal stress near to the average net normal stress of all experimental data used for fitting. As mentioned before this suggests dependency of ($\kappa$) on net normal stress. In order to investigate this dependency equation 4 is fitted to experimental shear strength of each net normal stress independently (equation 7).

$$\tau = \left(c' + \left(\sigma_n - u_a\right)\tan\phi'\right) + \left(\left(u_a - u_w\right)\left\{\Theta^{\kappa_\sigma} \cdot \tan\phi'\right\}\right) \quad (7)$$

where: $\kappa_\sigma$ is net normal stress dependent fitting parameter with values shown in table 3. Predicted shear strengths using equation 4 and 7 are then compared experimental data in figure 5(a, b and c). Since experimental data provided by Escario and Juca (1989) for Gray clay contains shear strengths under only one net normal stress the data published by Escario and Saez (1986) on same soil was used for that case. The experimental data of Escario and Saez (1986) are limited to suction range 0 to 800 kPa. For Red clay and Clayey sand the data of Escario and Juca (1989) were used.

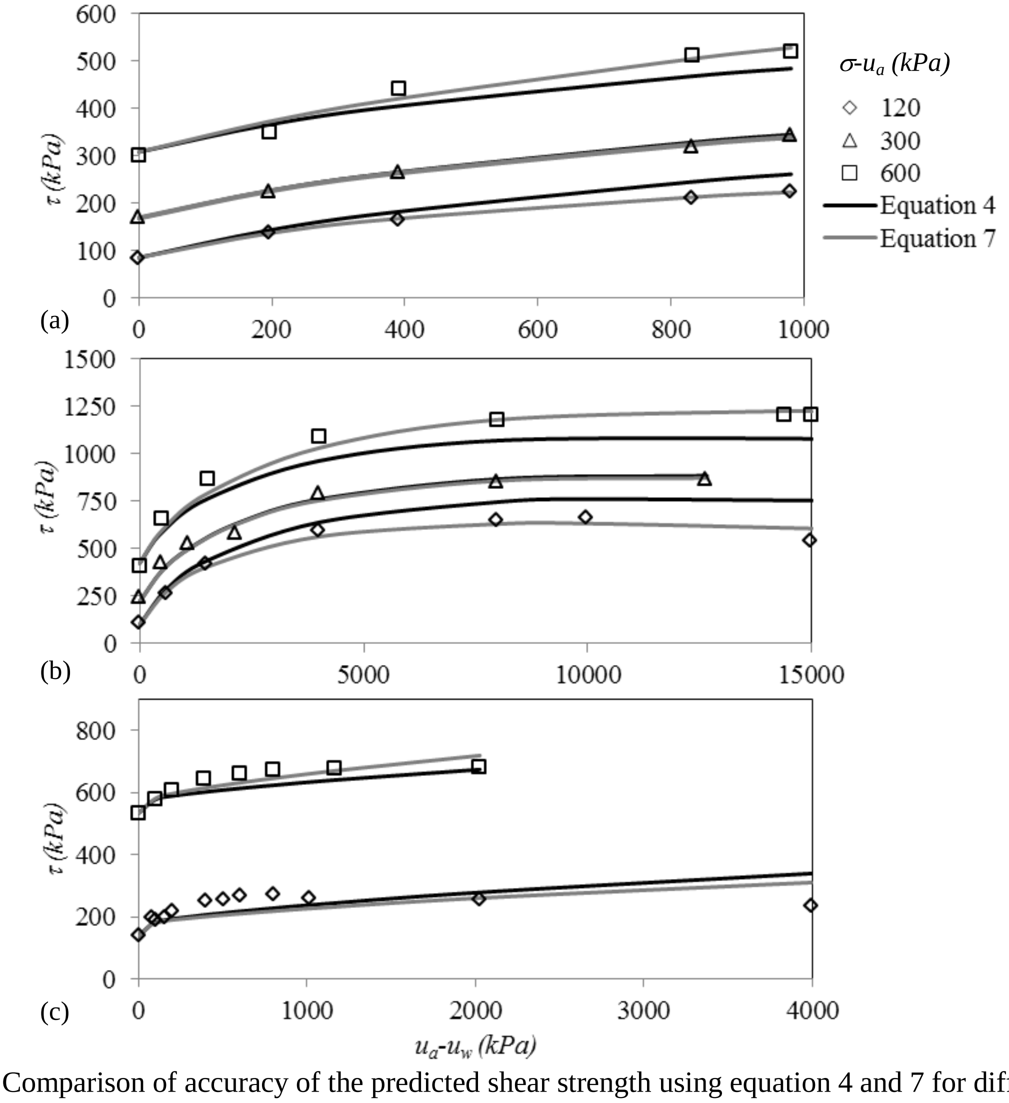


Figure 5. Comparison of accuracy of the predicted shear strength using equation 4 and 7 for different net normal stresses on: (a) Gray clay (Experimental data from Escario and Saez, 1986); (b) Red clay and (c) Clayey sand (Experimental data from Escario and Juca, 1989).

As presented in figure 5, a significant improvement in predicted shear strength is observed when using equation 7 in which ($\kappa_\sigma$) is used instead of ($\kappa$) of equation 4 except for figure 5.c with net normal stress equal to 120 kPa which the change is not as significant as the others. This observation shows dependency of ($\kappa$) to net normal stress along with other parameters (i.e. Plasticity Index-PI as suggested by Vanapalli and Fredlund, 2000).

Table 3. Net normal stress dependent fitting parameter ($\kappa_\sigma$) for equation 7.

| $\kappa_\sigma$ | | | $\sigma$-$u_a$ |
|---|---|---|---|
| Gray clay | Clayey sand | Red clay | (kPa) |
| 3.603893 | 2.066915 | 1.977061 | 120 |
| 2.978247 | - | 1.821677 | 300 |
| 2.218505 | 1.738256 | 1.674952 | 600 |

Thus, it should be noted that having a fitting parameter in a formula brings some limitations regardless of the improvements it provides. Formula needs additional experimental data from unsaturated tests to be adjusted which is not preferred in engineering practice. One way to avoid using experimental data in order to adjust the formula can be studying the parameters to which $\kappa_\sigma$ depends and formulating it based on these parameters. Figure 6.a and 6.b shows variation of $\kappa_\sigma$ with respect to net normal stress and PI respectively.

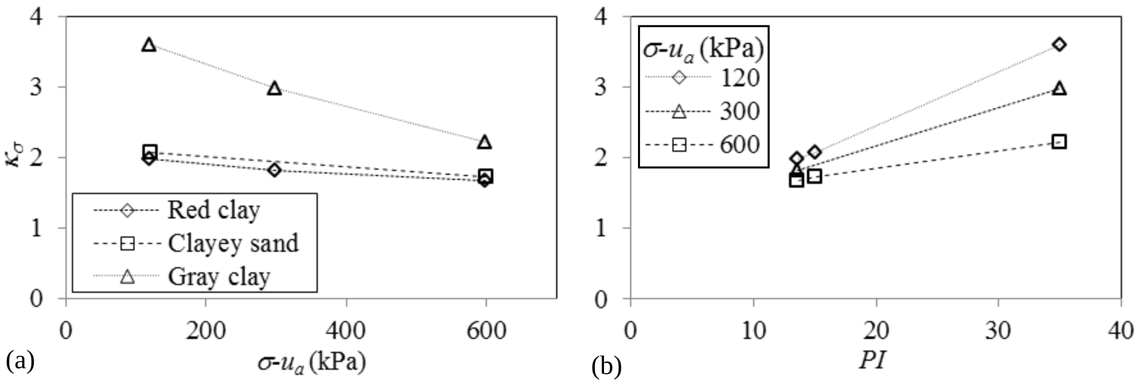


Figure 6. Variation of $\kappa_\sigma$ with respect to: (a) net normal stress and (b) PI.

As shown in figure 6.a, $\kappa_\sigma$ decreases with increasing net normal stress. However, the rate of decreasing $\kappa_\sigma$ is higher for soils with higher PI. For Clayey sand and Red clay that have very close PIs variation of $\kappa_\sigma$ with respect to net normal stress is almost same. Moreover, because their PI is very low, the variation is very small. Figure 6.b shows $\kappa_\sigma$ increases rapidly with increasing PI under lower net normal stresses. It is better shown in figure 6.b that $\kappa_\sigma$ for soils with lower PI is less dependent on net normal stress.

A multiple linear regression is conducted on parameters in order to build a relationship between obtained $\kappa_\sigma$ values and PI and net normal stress and the result is shown in equation 8 as follows:

$$\kappa_\sigma = C_1 + C_2 \cdot (\sigma_n - u_a) + C_3 \cdot PI + C_4 \cdot \left[(\sigma_n - u_a) \cdot PI\right] \quad (8)$$

where: $\sigma_n$-$u_a$ is in kPa and $C_i$ are constant and are equal to:

$$C_1 = 0.8369; C_2 = 0.0009; C_3 = 0.0876; C_4 = -0.0001$$

Equation 8 estimates the values of $\kappa_\sigma$ with less than 5% error. This precision shows the dependency of $\kappa_\sigma$ on net normal stress and PI is more significant than any other parameter.

## 6 CONCLUSIONS

Experimental data of Escario and Juca (1989) and also Escario and Saez (1986) are used to compare the precision of predicted shear strength of unsaturated soils by three famous formulae. It is observed that the best predictions are made by equation 4 suggested by Vanapalli et al. (1996) and Fredlund et al. (1996) because this formula contains a fitting parameter ($\kappa$) which other formulae do not. However, it is shown that the precision of this formula is highly dependent on net normal stress along with PI.

It is shown that this formula can be improved by replacing the parameter $\kappa$ with a net normal stress dependent parameter ($\kappa_\sigma$) in equation 7. The dependency of the new parameter ($\kappa_\sigma$) of equation 7 to PI to net normal stress is also investigated and a strong relationship that estimates $\kappa_\sigma$ as a function of PI and net normal stress with less than 5% error is established (equation 8). This in turn suggests that a similar study using more extensive experimental data on a variety of different soil types can lead to a more general relationship for $\kappa_\sigma$ in terms of net normal stress and PI. Such a relationship can replace costly and difficult experiments for estimating shear strength of unsaturated soils to some extent.

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