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# Determination of subgrade reaction modulus of two layered soil

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ABSTRACT: The subgrade reaction modulus, that indicates the relation between soil pressure and deflection, is one of the most efficient parameter that is used for structural analysis of foundation members. The popular practical method for estimating the modulus of subgrade reaction is the plate load test (PLT) that determines the ratio of load to displacement of circular plate with 15cm-75cm diameter. Afterward, the results modified with Terzaghi's equation (1955) and became applicable for foundation engineering problems on one-layer soil. The subgrade reaction modulus ($K_s$) depends to some parameters like soil type, size and shape of foundation, depth and stress level. In this paper the effect of soil layering on determination of subgrade reaction modulus ($K_s$) is investigated by using finite element analysis. For this purpose, the circular plate was considered on the different sand layer thickness with underlying clayey soil layer. The back analysis method is used to estimate the subgrade reaction modulus in each case. The results illustrate that as the thickness of sand layer increases, the modulus of subgrade reaction increases as well, until achieve maximum value in H/D=2 (H/D= ratio of sand layer thickness to plate diameter).

## 1 INTRODUCTION

Soil medium has very complex and erratic mechanical behavior, because of the Nonlinear, stress dependant, anisotropic and heterogeneous nature of it. Hence, instead of modeling the subsoil in its three dimensional nature, subgrade is replaced by a much simpler system called a subgrade model that dates back to the nineteenth century. The search in this context leads to two basic approaches which are Winkler approach and the elastic continuum model are of widespread use, both in theory and engineering practice. Winkler (1867) assumed the soil medium as a system of identical but mutually independent, closely spaced, discrete and linearly elastic springs and ratio between contact pressure, *p*, at any given point and settlement, *w*, produced by load application at that point, is given by the coefficient of subgrade reaction, $K_s$ :

$$K_s = p / w \quad (1)$$

A direct method to estimate both $E_s$ and $K_s$ is plate loading test (PLT) that requires circular or equivalent rectangular plates. PLT provides a direct measurement of the compressibility and bearing capacity of soil and essentially consists in loading a rigid plate and determining the settlements corresponding to each load increment. The results of a PLT are presented as applied contact pressure versus settlement curves (Figure 1).

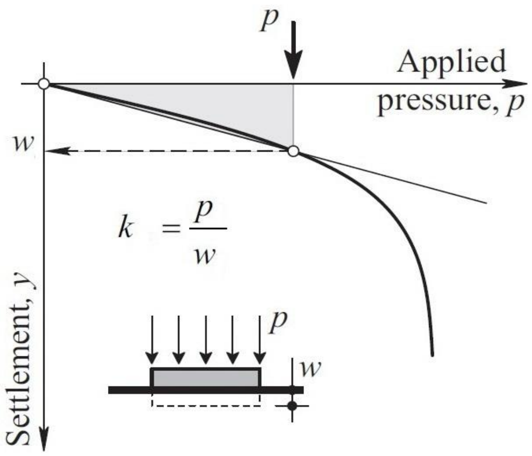



Figure 1. Typical presentation of results from a PLT (Teodoru and Toma 2009).

The coefficient of subgrade reaction, *Ks*, is the initial slope of the curve (Fig. 1) until the limit pressure, *p*, is reached.

Many researches including Biot (1937), Terzaghi (1955), Vesic (1961) and have investigated the effective factors and determination approaches of *Ks*. Geometry and dimensions of the foundation and soil layering are assigned to be the most important effective parameters on *Ks*.

Terzaghi (1955) made some recommendations where he suggested values of $K_s$ for 1*1 ft rigid slab placed on a soil medium; however, the implementation or procedure to compute a value of K for use in a larger slab was not specific. Biot (1937) solved the problem for an infinite beam with a concentrated load resting on a 3D elastic soil continuum. He found a correlation of the continuum elastic theory and Winkler model where the maximum moments in the beam are equated. Vesic (1961) tried o develop a value for $K_s$, except, instead of matching bending moments. He matched the maximum displacement of the beam in both models. He obtained the equation for $K_s$ for use in the Winkler model.

Several studies by Filonenko-Borodich (1940), Heteneyi (1950), and Pasternak (1954), and others, have attempted to make the Winkler model more realistic by assuming some form of interaction among the spring elements that represent the soil continuum.

One of the early contributions was that of Terzaghi (1955), who proposed that $K_s$ for full-sized footings could be obtained from plate-load tests using the following equations:
For clayey soil:

$$Ks = Kp\frac{Bp}{B} \tag{4}$$

For sandy soil:

$$Ks = Kp\left(\frac{B + Bp}{2B}\right)^2 \tag{5}$$

Where

Ks = desired value of modulus of subgrade reaction for the full-size (or prototype) foundation
$K_p$= value obtained from a plate-load test using a 0.3m X 0.3 m (1ft X 1 ft) or other size load plate
$B_p$= plate diameter
B= dimension of foundation

In the present paper, the effect of soil layering on $K_s$ determination was investigated. For this purpose, the numerical analysis are performed on different sand layer thickness with underlying clayey soil layer and the subgrade reaction modulus are determined using back analysis method.

## 2 ANALYSIS PROCEDURE

*A.Geometry*

78 finite element analyses performed using Plaxis 8.5 software with axis-symmetric model. The 15-node triangular elements are used to model the soil layers and other volume clusters. Near the edges of a loaded area where stress concentrations are expected, mesh is refined by reducing the size of the soil surface below the position of the loading area. Analysis is performed under load control by a vertical distributed load boundary condition applied to the soil surface below the position of the loading plate. It is assumed that both horizontal and vertical displacements are zero for all nodes along the bottom boundary of the mesh. On the vertical side boundaries, the horizontal displacements have been assumed to be zero too.

The size of axis-symmetric model in X direction is considered as 2.5D where D is foundation diameters and 6D in Y direction based on Bowles (1997) recommendation.

The depth of upper layer (sand) is coefficient of diameter of foundation (Figure 2). This (n= H/D) coefficient varies from 0.25, 0.5, 1, 1.5 to 3.5. Afterward, all of these models were analyzed under the same incremental loads. Then the stress-settlement curves are drawn and the subgrade reaction modulus for each model is obtained (Figure 6).

*B.Material properties*

*Sand*

In this paper, the sand layer is considered as Hardening-Soil model. Schanz et al (1998, 2000) presented the hardening soil model for simulating the behavior of different types of soil. In the special case of a drained triaxial test, the observed relationship between the axial strain and deviatoric stress can be approximated by hyperbola. Such a relationship was first formulated by Kondner (1963) and later used in the well-known hyperbolic model (Duncan & Chang, 1970). In current study, the Hostun sand properties (Schanz et al. 1997) are considered for upper sand layer (Table 1).

*Clay*

In this study the material properties of Haney clay that were provided by Vaid and Campanella (1977) are dedicated to clay layer. The Haney clay properties are shown in Table 2.

Table 1. Loose Hostun sand properties (Schanz et al. 1997).

| $\gamma$ (kN/m$^3$) | $E_{50}^{ref}$ (kN/m$^2$) | $E_{ur}^{ref}$ (kN/m$^2$) | $E_{oed}^{ref}$ (kN/m$^2$) | $\varphi$ ($^o$) | $\psi$ ($^o$) | C (kN/m$^2$) | $K_0^{NC}$ | $\upsilon_{ur}$ | *m* | $R_f$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Unit weight | Secant stiffness | Unloading /reloading stiffness | Tangant stiffness | Friction angle | Dilatancy angle | Cohesion | Coefficient of lateral earth pressure | Unloading -reloading Poisson's ratio | Power for stress-level | Failure ratio |
| 17 | 20000 | 60000 | 16000 | 34 | 0 | 0 | 0.44 | 0.2 | 0.65 | 0.9 |

Table 2. Haney clay properties (Vaid & Campanella 1977).

| $k_m$ | $\lambda_m$ | $\mu_m$ | $\varphi$ ($^o$) | $\psi$ ($^o$) | C (kN/m$^2$) | $K_0^{NC}$ | $\upsilon_{ur}$ | $K_x$,$K_y$ (m/day) |
|---|---|---|---|---|---|---|---|---|
| Modified swelling index | Modified compression index | Modified creep index | Friction angle | Dilatancy angle | Cohesion | Coefficient of lateral earth pressure | Poisson's ratio | permeability |
| 0.016 | 0.105 | 0.004 | 32 | 0 | 0 | Default | 0.15 | 0.001 |

## 3 RESULTS

The models analyzed and the settlements are measured. Figures 3 and 4 show the vertical displacements and effective stress distribution of sample with H/D=3.5, respectively. The pressure-settlement curves for all models with various ratio of H/D are shown in Figure 5. The subgrade reaction modulus for each model calculated by estimation inverse slope of linearly part of these curves. Due to obtained results the variation of $K_s$ versus H/D ratio are presented in Figure 6.

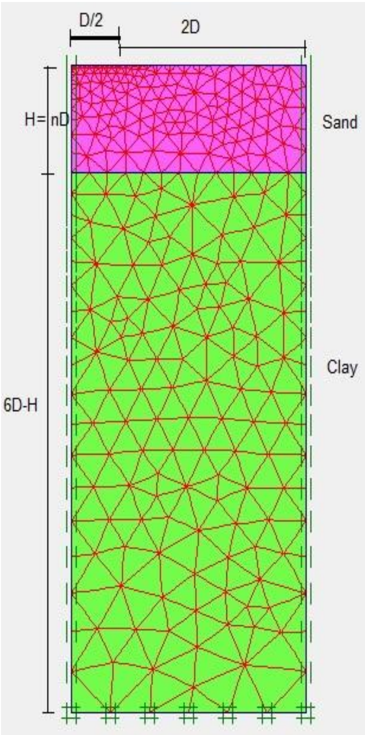



Figure 2. Model geometry.

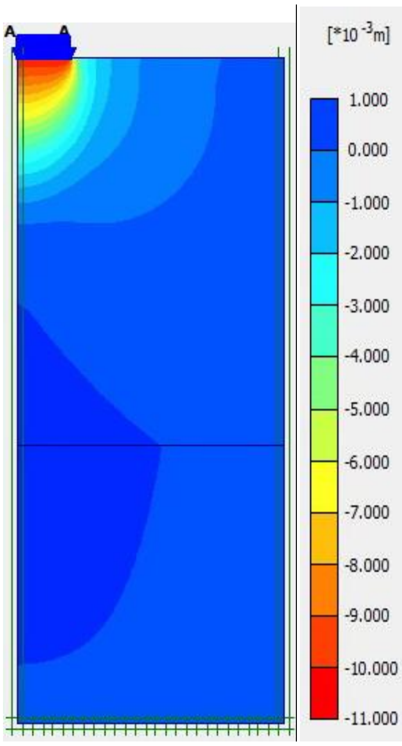



Figure 3.Vertical displacements.

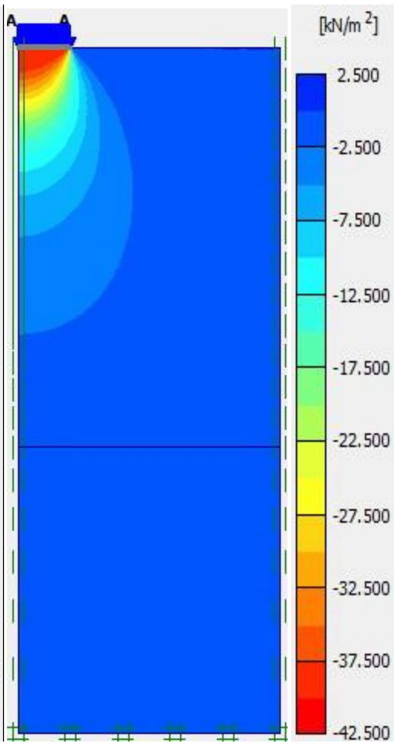
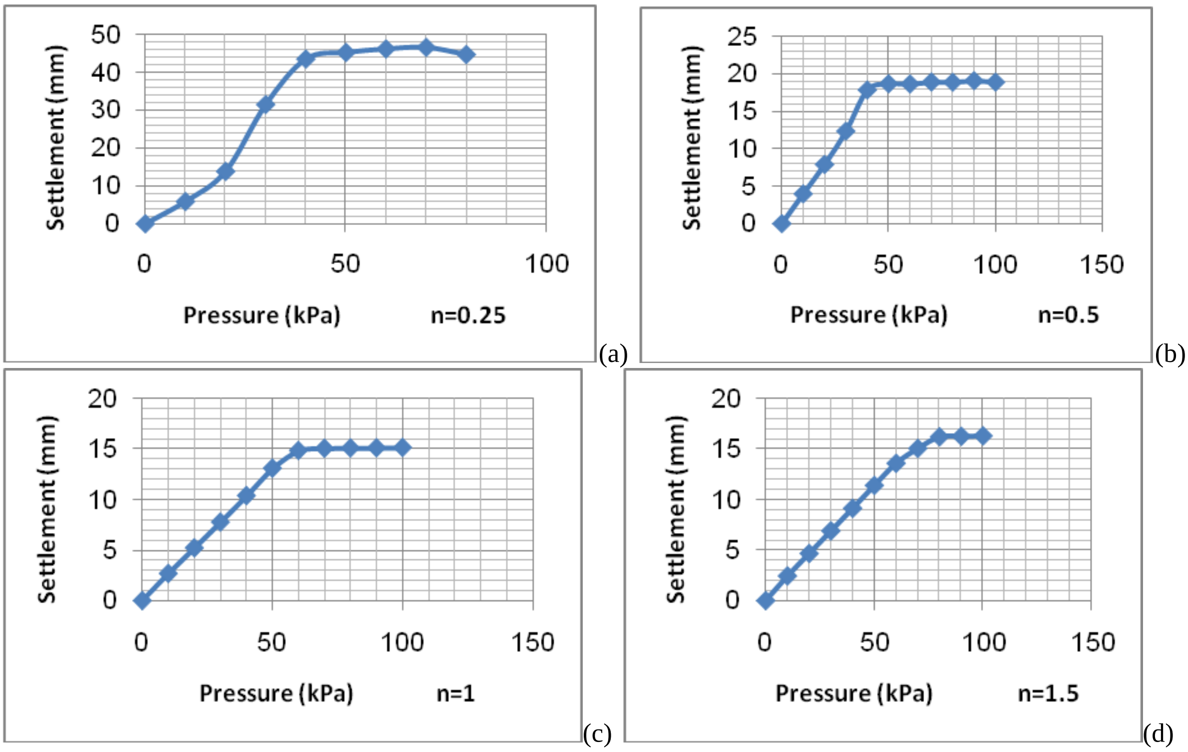
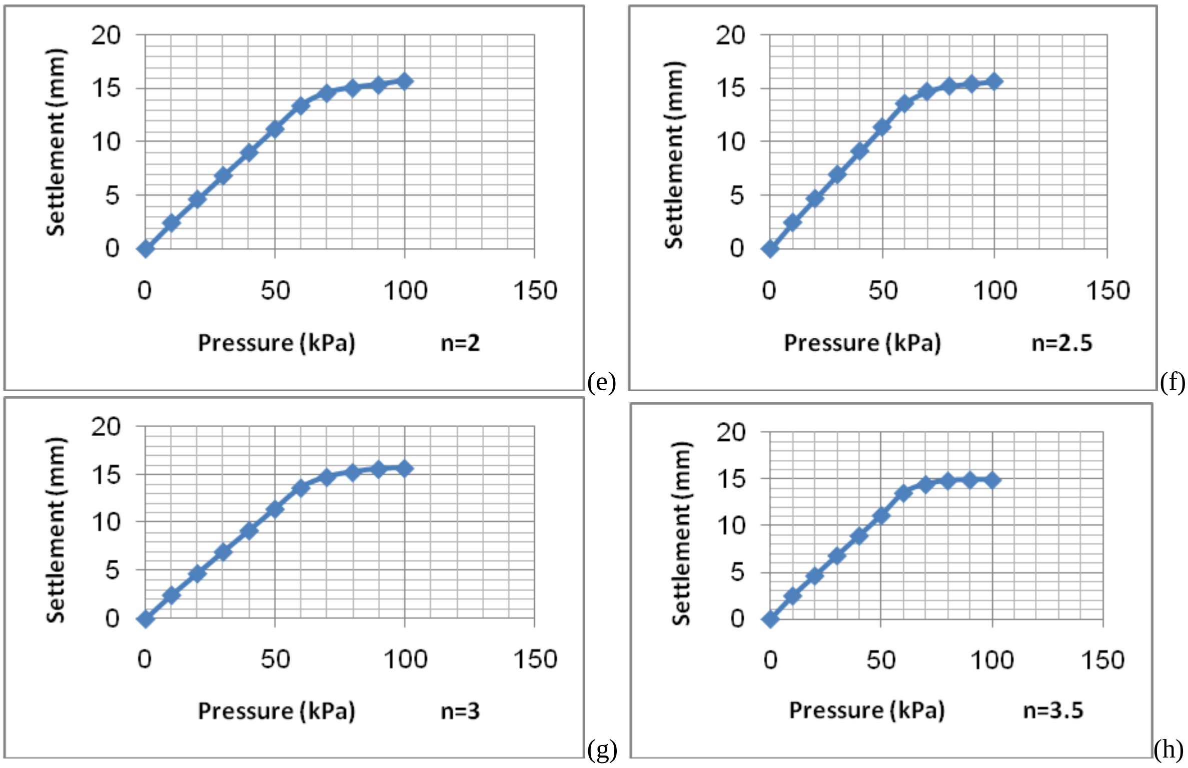



Figure 4. Vertical stresses.






Figures 5. Pressure-settlement curves for models with various ratio of n= H/D.

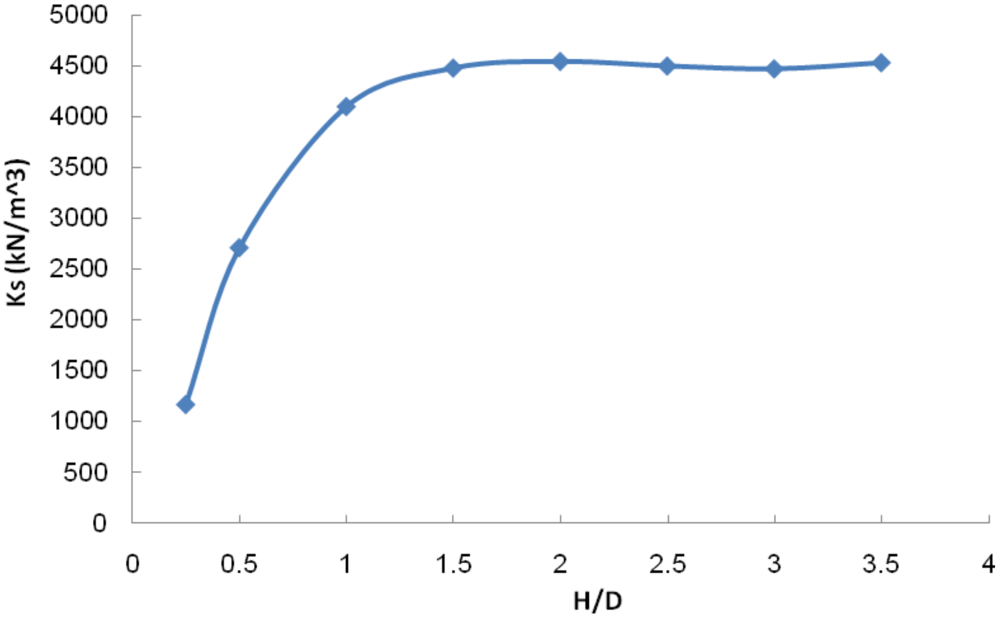



Figure 6. Subgrade reaction modulus for different ratio of H/D.

## 4 CONCLUSIONS

In this paper the effect of soil layering on determination of subgrade reaction modulus ($K_s$) is investigated by using finite element analysis. For this purpose, the circular foundation was considered on different sand layer thickness with underlying clayey soil layer and the subgrade reaction modulus are calculated by back analysis method. The results illustrate that as the sand layer thickness increases, the modulus of subgrade reaction increases as well, until achieve maximum value in H/D= 2, and after that the effect of sand layer thickness on $K_s$ determination becomes insignificant. Mentioned results indicate that clayey soil can play notable rule in subgrade reaction modulus determination, especially in great mat foundations that their dimensions are considerable compare to sand layer thickness.

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