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Nonlinear analysis of laterally loaded piles

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ABSTRACT : The problems of the analysis for laterally loaded single isolated piles are considered. Mechanical model for the pile-soil allowing nonlinear relation between active lateral loads and resulting lateral displacements is described. Nonlinear analysis of piles for lateral load, taking the vertical component effect of external load into account, has been developed on the basis of Winkler's hypothesis for piles of fixed cross-section and any length –from absolutely rigid to infinitely long- and is compared with statistical test data.

1 INTRODUCTION

The analysis of laterally loaded piles is one of the most complicated problems in soil mechanics. It comprises several aspects, each representing in itself great difficulties. Firstly, the curve showing lateral displacements of piles vs active lateral loads has an evident nonlinear pattern. Secondly, the same laterally loaded pile under the same soil conditions conforms to the different structural models, and can behave as a structure of finite rigidity, an absolutely rigid or slender (infinitely long) structure, depending on the relative depth of driving. Experimental data show that at particular driving depth the pile could behave as a slender structure for a low values of lateral displacements and as a structure of finite rigidity for large values of lateral displacements. The pile of finite rigidity, in its turn, can approach the absolutely rigid pile by the pattern of its behavior in soil at large displacements. Thirdly, soil, in fact, is a laminated base having different strength and deformation characteristics. Fourthly, the influence of the external load vertical component on the resistance of the pile to lateral loads is ambiguous and could produce both positive and negative effects.
There are solutions for the particular problems in the worldwide practice, i.e. for absolutely rigid piles, piles of finite rigidity and for slender piles which have not got yet a strict differentiation and this complicates their practical usage since the existing methods of analysis lead to serious errors especially at the boundaries of structural models.
The wide usage of statistical test methods as for estimating actual bearing capacity and stress-strain behavior of vertically loaded piles does not solve the problem of determining actual resistance of piles to lateral loads to a full measure. Test results allow only to determine the relation of lateral displacements and lateral loads applied to piles at some particular height above the soil surface. The distribution of internal stresses, shear stresses and bending moments necessary for the choice of longitudinal and transverse reinforcement remains unknown as well as the resistance of a pile to the horizontal stress imposed at vertical height above the soil surface in the presence of vertical component.
The variation of these parameters complicates the test procedure considerably and significantly affects the cost in the case of field tests.
The author who has been tackling the problem about thirty years has managed to acquire a representative enough information on single isolated piles of different structure and groups of piles combined by a rigid foundation mat, tested under different soil conditions. For single piles and their groups combined by rigid

foundation mat new methods of analyzing the stress-strain behavior under lateral and vertical loads and moments have been developed on the basis of the acquired information and with the usage of the strain-gauge test methods.

2 ANALYSIS

The analysis of laterally loaded single isolated piles in a uniform soil is considered below. The soil represented according to Winkler's hypothesis has the bedding value varying with the lateral displacement, embedding depth and slenderness of a pile.

$$K_{(x)} = K_{(z)} \left(\frac{x}{z}\right)^{\nu} \quad (1)$$

Where K(x) is a bedding value for soil at the depth x, K(z) is a bedding value for soil at the pile tip level, z is an embedding depth of a pile and ν is a diagram transformation coefficient for K(x), showing the plastic strain development in soil embedding the pile and depending on the lateral displacement value and the relative slenderness Fe of a pile, formule (7a).

In the experimental investigation at the initial stage of loading (at lateral displacement of the order 0,001-0,002) it was found that the lateral load-displacement relation is of a linear nature and the reactive resistance for soil is directly proportional to the elastic line (or bend line) of an absolutely rigid pile, having the maximum ordinate at the soil surface level. For the given range of displacements when the structural strength of the soil has not been yet overcome the soil bedding value is constant for the depth (ν=0). For x=0 and ν=0 the indeterminate form 0^0 appears in the equation 1 and its evaluation shows that $\left(\frac{x}{z}\right)^{\nu}$ tends to 1.

The linear relation is violated owing to the appearance of plastic strains and soil failure, extending from above downwards under further increase in loading and lateral displacements. At the surface the soil tends to zero causing modification in the bedding value law as to the depth. The bedding value remains constant only in the boundaries of those parts of the pile where displacements do not produce plastic strains in the surrounding soil (ν>0). The location of the zero point Z0 as well as that the maximum ordinate of the q(x) diagram (fig.1) is changing with the development of plastic strains and subsequent failure of soil, capturing an ever-growing region extending from the soil surface downwards. The above outlined mechanism accounts for the physical essence of the relation between the lateral load applied to the pile and lateral displacements produced by this load.

It should be added that the bending stiffness of a reinforced pile cross-section is a variable quantity and a function of u_0 since the amount of cracking in the pile is related to the bending moment value which is a function of u_0 . the nonlinearity of the relation $Q = f(u_0)$ is also evident due to the variable nature of the inertia moment of a pile cross-section, J, related to the cracking which could appears when transporting and driving the pile.

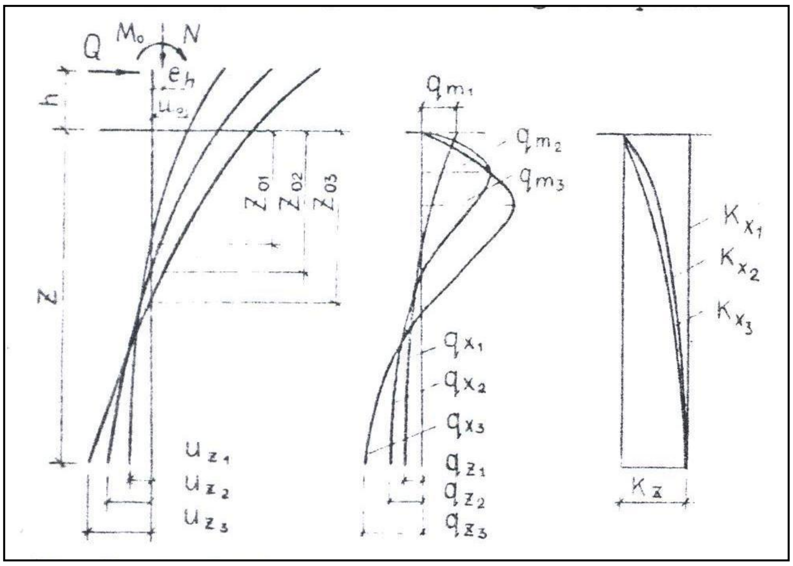

Figure 1. Analysis diagram for a laterally loaded pile.

Experimental investigation on different piles under different soil conditions have given evidence of the friction forces developing along the sides of the pile that should not be neglected. The ratio of the integral characteristics of forces developing along the face and side planes of the pile is not the same: it has greater value at the beginning and small value at the end of the loading. The value of this ratio depends on the capacity of the soil to bear tangential stress. However, the possibility arises to take the integral characteristics of friction forces into account through increasing the normal forces developing on face planes by an appropriate quantity while holding overall equilibrium conditions of the pile-soil system since these characteristics are practically independent of the side area and depend only on normal pressures forces (compression). This procedure is quite justified and the results of the theoretical analysis agree with the experimental data for different piles(f.i. of round and rectangular cross-sections).

The value of the plastic strain development coefficient ν and those of the soil bedding value K(z) reduce with the increase in slenderness when the pile embedding depth changes and consequently causes the change of slenderness of the pile-sol system. for absolutely rigid piles the K(z) values are quite independent of the pile embedding depth although the K(x) values is variable for the limits of the pile length. It is very important since for this case K(z) is an ambiguous value and there appears the possibility to form the suitable tables for K(z) values and to take their variation with relative slenderness of the pile into account.

Experimental investigations have shown the ambiguity of the vertical component effect on lateral load resistance of the pile. The vertical component exerts a positive effect by holding pile of lateral displacements at the initial loading stage when axial vertical force is constant. The effect becomes negative when the vertical component action line goes beyond the pile cross-section core due to the development of lateral displacement with the increase in lateral loading. The influence of the vertical component could have only negative or only positive effects for the whole range of lateral displacements depending on the eccentricity of the vertical force application as to geometric axis of the pile (±eh).

Proceeding from the above the expressions have been obtained for the analysis of single isolated piles under simultaneous lateral and vertical loading.
The main expressions are given below.

The expression for lateral force resistance of a pile has the form:

$$Q = BK_{(z)}zu_0\left[\frac{1}{\nu+1} - \frac{1}{(\nu+2)\alpha} - \frac{F_eF_{\alpha 2}}{\alpha(1-\alpha+F_eF_{\alpha 1})}\right] \quad (2)$$

The shear force in an arbitrary cross-section x could be written as:

$$Q_{(x)} = Q - BK_{(z)}zu_0\left[\frac{1}{\nu+1}\left(\frac{x}{z}\right)^{\nu+1} - \frac{1}{(\nu+2)\alpha}\left(\frac{x}{z}\right)^{\nu+2} - \frac{F_e(1-\alpha)}{\alpha(1-\alpha+F_eF_{\alpha 1})}\left\{A_1\left(\frac{x}{z}\right)^{\nu+2} + A_2\left(\frac{x}{z}\right)^{\nu+3} - A_3\left(\frac{x}{z}\right)^{\nu+4} +\right.\right.$$

A4xzν+6−A5xzν+7 (3)

The bending moment in an arbitrary cross-section x is:

$$M_{(x)} = M_0 + Q_{(h+x)} + N\left(u_{(h)} + u_{(z)} + e_{(h)} - \frac{B}{2}\right) + BK_{(z)}z^2u_0\left[\frac{1}{(\nu+1)(\nu+2)}\left(\frac{x}{z}\right)^{\nu+2} - \frac{1}{(\nu+3)(\nu+2)\alpha}\left(\frac{x}{z}\right)^{\nu+3} -\right.$$

Fe(1−α)α(1−α+FeFα1)A1ν+3xzν+3+A2ν+4xzν+4−A3ν+5xzν+5−A4ν+7xzν+7−A5ν+8xzν+ 8 (4)

The pile cross-section bend angle at the soil surface level is expressed as:

$$\varphi_{(0)} = \frac{u_0}{\alpha z}\left(1 + \frac{F_e(1-\alpha)}{1-\alpha+F_eF_{\alpha 1}}F_{\alpha 3}\right) \quad (5)$$

The pile tip displacements is given by:

$$u_{(z)} = \frac{u_0(1-\alpha)^2}{\alpha(1-\alpha+F_eF_{\alpha 1})} \quad (6)$$

The relative depth of the pile zero point location α=z0/z is defined from the equation 7:

$$\frac{(\lambda+1)(\nu+2)-1}{(\nu+1)(\nu+2)} - \frac{(\lambda+1)(\nu+3)-1}{(\nu+2)(\nu+3)\alpha} - \frac{F_e(1-\alpha)}{\alpha(1-\alpha+F_eF_{\alpha 1})}F_\nu + \frac{M_0}{z} = 0 \quad (7)$$

Where $F_e = BK_z z^4/360EJ$; (7a)

$$F_{\alpha 1} = 26 - 33\alpha - F_{\alpha 3} \quad (8)$$

$$F_{\alpha 2} = A_1 + A_2 - A_3 + A_4 - A_5 \quad (9)$$

$$F_{\alpha 3} = 45\alpha - 80\alpha^2 + 30\alpha^3 - 2\alpha^5 \quad (10)$$

$$A_1 = \frac{F_{\alpha 3}}{\nu} + 2 \; ; \; A_2 = \frac{60\alpha-45}{\nu+3} \; ; \; A_3 = \frac{30\alpha-20}{\nu+4} \; ; \; A_4 = \frac{3\alpha}{\nu+6} \; ; \; A_5 = 1/(\nu+7) \quad (11)$$

$$F_\nu = F_{\nu 1} + F_{\nu 2} - F_{\nu 3} + F_{\nu 4} - F_{\nu 5} \quad (12)$$

$$F_{\nu 1} = \frac{A_1((\lambda+1)(\nu+3)-1)}{\nu+3} \quad (13)$$

$$F_{\nu 2} = \frac{A_2((\lambda+1)(\nu+4)-1)}{\nu+4} \quad (14)$$

$$F_{\nu 3} = \frac{A_3((\lambda+1)(\nu+5)-1)}{\nu+5} \quad (15)$$

$$F_{\nu 4} = \frac{A_4((\lambda+1)(\nu+5)-1)}{\nu+7} \quad (16)$$

$$F_{\nu 6} = \frac{A_6((\lambda+1)(\nu+5)-1)}{\nu+8} \quad (17)$$

The remaining notations are given in Figure 1.

Soil bedding values Kz and plastic strain development coefficients ν are tabulated as functions of soil type, soil state, the slenderness Fe and of lateral displacements of the pile at the soil surface level.

Figure 2 shows the results of the analysis with the usage of a computer for piles of 0,3*0,3m (b=0.3m) cross-section driven to the depth z=2, 4 and 6m at h=0 into a sandy soil having yield ratio JL=0.4

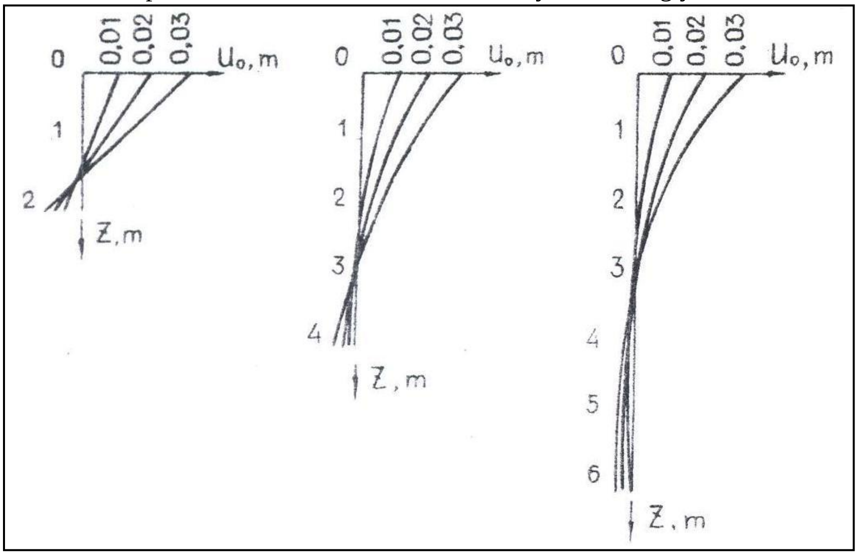

Figure 2. the elastic lines of piles.

Figure 3, 4 and table 1 show the calculation load-displacement and load-driving depth plots, and the calculation results, respectively, in comparison with experimental data.

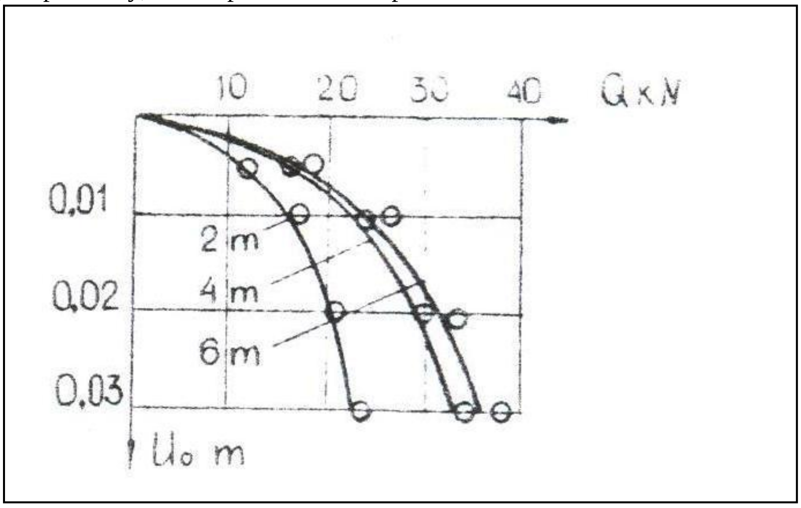

Figure 3. The load-displacement plots.

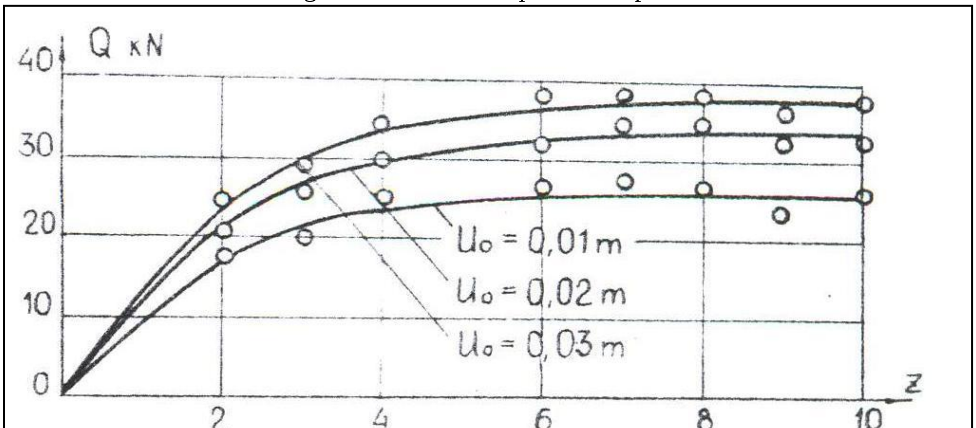

Figure 4: The resistance variation vs driving depth in comparison

with the statistical test data: (-) calculated; (o) experimental.

Table.1: calculated results for laterally loaded piles

Z m	U0 m	α	Uz m	Kz KH/m³	Q KN
2	0.01	0.693	0.0039	20000	16.3
	0.02	0.707	0.0071		28.5
	0.03	0.739	0.0098		22.3
4	0.01	0.613	0.0016	18950	22.8
	0.02	0.658	0.0032		29.4
	0.03	0.688	0.0049		33.0
6	0.01	0.505	0.000043	13860	24.5
	0.02	0.538	0.00087		31.6
	0.03	0.560	0.0013		35.9

The plots give evidence of the satisfying agreement between the calculated results and test data. In case of an absolutely rigid pile the analysis formula (2), (7) are simplified for EJ=oo, reducing to the particular case when bending could be neglected, considered in the ref.(Shakhirev 1987).

3 CONCLUSIONS

The developed method of analysis has been tested and has given positive results in designing pile supports for heating networks pipelines at the number of industrial projects.

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