Numerical evaluation of the bearing capacity factor N'$_\gamma$ of ring footings for associated soils

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ABSTRACT: The aim of this work is to carry out numerical computations using FLAC (Fast Lagrangian Analysis of Continua) to evaluate the soil-bearing capacity factors N'$_\gamma$ for both smooth and rough ring footings, for low and high friction soils (φ = 20° to 45°). The results indicate a significant decrease in the value of N'$_\gamma$ with an increase in the ratio of internal radius to external radius of the ring. The values of N'$_\gamma$ for a rough footing base, especially with greater values of φ, are shown to be significantly higher than those for a smooth footing base. The computational results are presented in the form of design tables and graphs and compared with previous experimental and numerical published results available in the literature.

1 INTRODUCTION

Nowadays, ring footings are often provided for structures such as silos, storage tanks, chimneys, and bridge piers. The use of ring footings decrease the amount of material used and is more economical. Safe and economical design of such footings requires a good knowledge of both the settlement and bearing capacity relating to ring footings. The bearing capacity of strip and circular footings has been extensively studied for many decades. However, very few attempts have been made to study the bearing capacity of ring footings. A few experiments have also been performed to determine the bearing capacity of ring foundations (Saha, 1978; Saran et al., 2003). By using the method of characteristics, Kumar and Ghosh (2005) have obtained the bearing capacity factor N'$_\gamma$ for both smooth and rough ring footings assuming that the interface friction angle between the footing base and the underlying soil mass increases gradually from zero along the axis of symmetry to φ along the outer edge of the footing. Whereas, Boushehrian and Hataf (2003) carried out a finite element analysis to determine the load - deformation response of rigid ring footings. By using the finite difference method, Zhao and Wang (2008) have computed N'$_\gamma$ for low friction soils (φ = 5° to 30°). Recently Choobbasti et al. (2010) have performed a numerical study using Plaxis software to evaluate the bearing capacity and settlement of a ring footing for a friction soil of φ = 26°.
The aim of this work is to carry out numerical computations using FLAC (Fast Lagrangian Analysis of Continua) (2005), to evaluate the soil-bearing capacity factors N'$_\gamma$ for both smooth and

rough ring footings, for low and high friction soils (φ = 20° to 45°). Then, the computational results are compared with previous published results available in the literature.

2 OVERVIEW OF PREVIOUS WORK

The bearing capacity of a shallow strip footing is commonly determined by using the Terzaghi formula:

$$q_u = cN_c + qN_q + \frac{1}{2} B\gamma N_\gamma \tag{1}$$

Where q_u is the ultimate bearing capacity, c is the soil cohesion, q is the surcharge above the base level of the footing, γ is the soil unit weight, B is the footing width, N_c, N_q, N_γ are the bearing capacity factors representing the effect of cohesion c, surcharge q and unit weight γ respectively.

There are several methods in the literature for the evaluation of N_γ, which are the limit equilibrium method (Terzaghi, 1943; Meyerhof, 1963; Meyerrhof, 1953), the limit analysis method (Chen, 1975; Michalowski, 1997; Soubra, 1999), the method of characteristics (Sokolowski, 1997; Martin, 2005) and the finite element method (Griffiths, 1982; Frydman, 1997).

The equation (1) is used for a strip footing, but, for another footing other than strip shape, the generalized bearing capacity equation is given as follows:

$$q_u = s_c cN_c + s_q qN_q + s_\gamma \frac{1}{2} B\gamma N_\gamma \tag{2}$$

Where s_c, s_q, s_γ are called shape factors. There are various values of these shape factors suggested in an empirical and semi-empirical ways. The principal authors having proposed these factors are Terzaghi, Caquot and Kérisel (1953), Hansen (1961), Meyerhof (1963) and De Beer (1970).

Circular footings were first studied by Shield (1955), Eason and Shield (1960), Cox et al. (1961), and later by, Bolton and Lau (1993), Cassidy and Houlsby (2002), Martin (2005). All these authors used the method of characteristics. The method of finite element was used by Manoharan and Dagbusta (1995), Loukidis and Salgado [(2009) whereas Erickson and Drescher (2002) used a finite difference method.

Axis-symmetric problems applicable to ring footing were studied by Kumar and Ghosh (2005) using the method of characteristics, which assumes a soil following an associated flow rule. The same assumption was employed by Zhao and Wang (2008) using the finite difference method. However, frictional soils are found experimentally to dilate at increments considerably less than those predicted by the normality condition, that is $\psi < \varphi$. Hence, real soils do not obey the associative flow rule. Indeed, values of bearing capacity factors for a non-associated flow rule ($\psi < \varphi$) may be significantly lower when φ is greater than about 30° (Griffiths, 1982; Frydman and Burd, 1997 ; Erickson and Drescher, 2002).

To take into account the effect of the nonassociativity, some authors (Drescher and Detournay, 1993); Michalowski and Shi, 1995) suggest modifying the values of c and φ by c* and φ* respectively as follows:

$$\tan\varphi^* = \frac{\cos\varphi\cos\psi}{1-\sin\varphi\sin\psi}\tan\varphi \tag{3}$$

$$c^* = \frac{\cos\varphi\cos\psi}{1-\sin\varphi\sin\psi}c \tag{4}$$

3 NUMERICAL MODELLING PROCEDURE

This paper deals with the numerical study of bearing capacity of smooth and rough rigid ring footings with internal and external radius r_i and r_0 respectively. The footing is subjected to an axial static load, and located on the surface of a cohesionless frictional associated soil. Since the problem is axis-symmetric, only half of the problem domain is considered. The computations have been done for the values of the r_i/r_0 ratio 0, 0.25, 0.33, 0.50 and 0.75. The idea is to sweep all ring footings cases, that is from the circular footing ($r_i/r_0 = 0$) until the unlikely ring footing ($r_i/r_0 = 0.75$).

The vertical and bottom boundaries were located at a distance of $8r_0$ and 16 r_0 respectively in order to minimise boundary effects. The bottom boundary was assumed to be fixed, and the vertical boundaries were constrained in motion in the horizontal direction as shown in Fig. 1.

The analysis is carried out using the computer code FLAC which is a commercially available finite difference explicit program. With this program, the solution of a static problem is obtained by including dynamic equations of motion. Damping terms are included to gradually remove the kinetic energy from the system. The software uses an explicit time-marching in which the stresses and deformations are calculated at several small time steps until a steady state is achieved in a numerically stable way. The code is most effective and appropriate when applied to nonlinear problems, or to situations in which physical instability may occur.

The elastic perfectly plastic Mohr Coulomb model encoded in FLAC is used. Physical and mechanical characteristics used in the present study are: a shear modulus $G = 10$ MPa, an elastic bulk modulus $K = 20$ MPa, a soil unit weight $\gamma = 20$ KN/m^3, and a series of six values of the angle of soil internal friction $\varphi = 20°$ to 45° with an increment of 5°.

In order to develop an acceptable analysis scheme for later computations, preliminary simulations have been carried out, by testing the size of the domain, the grid, and the boundary conditions. In the vicinity of the footing, the grid is refined to capture the large gradients in strain. The highest strain gradient will be in the region adjacent to the left and right sides of the part of the footing located between the internal radius and the external radius. The grid is therefore very fine in this area.

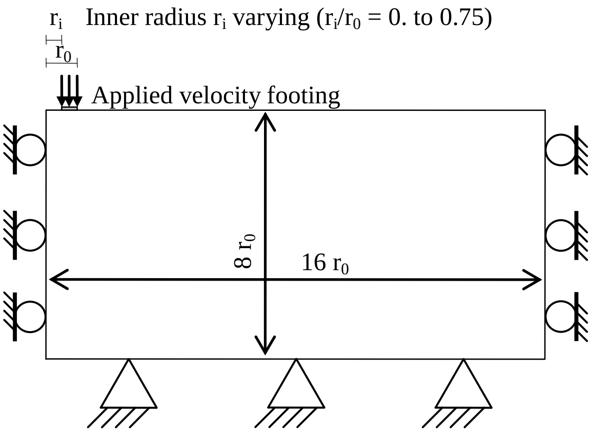


Figure 1. Model boundary conditions.

The proposed modeling procedure of the bearing capacity factors follows two steps. In the first one, the geostatic stresses are computed assuming the soil to be elastic. At this stage, some stepping is required to bring the model to equilibrium. In the second step, a downward velocity was applied to the gridpoints representing the footing. First, a relatively high velocity of 10^{-6} m/step is applied to the footing area, until a steady plastic flow is achieved (i.e. until a constant pressure is realized). As the level of errors in such calculation scheme depends on the applied velocity, a more accurate bearing capacity can be obtained by reducing the footing velocity by half, and continuing to a new steady plastic flow state. This procedure recommended by FLAC manual, is repeated several times, particularly for high values of the soil internal friction, until the difference between bearing capacity computed at two successive steady plastic flow states becomes negligible.

A rough ring footing was simulated by fixing the displacement in the radius direction to zero for the gridpoints representing the footing. The smooth ring footing was simulated by leaving the gridpoints free to move in the radius direction.

4 RESULTS OF COMPUTATION AND DISCUSSION

The bearing capacity factor N'_{γ} is dependent on the soil unit weight γ and was calculated assuming cohesionless soil ($c = 0$) with no surcharge ($q = 0$). Thus the generalized bearing capacity equation (2) becomes as follows:

$$q_u = \frac{1}{2} B \gamma s_{\gamma} N_{\gamma} \quad = \quad \frac{1}{2} B \gamma N'_{\gamma} \tag{5}$$

$$q_u = r_0 \gamma N'_{\gamma} \tag{6}$$

Where N'_{γ} is the bearing capacity factor for the ring footing.

The values of the bearing capacity factor N'_{γ}, are listed in table 1 for r_i/r_0 = 0, 0.25, 0.33, 0.5, 0.75, which shows the variation of N'_{γ} with φ, and r_i/r_0, for smooth and rough ring footings.

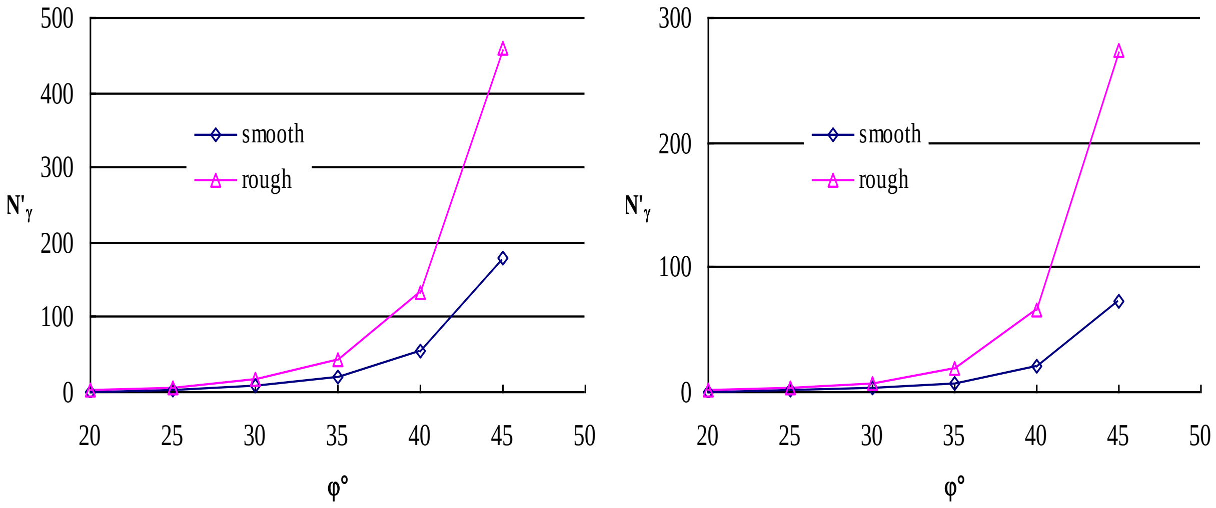
Fig. 2 shows the variation of N'_{γ} with φ respectively for circular footing ($r_i/r_0 = 0$) and for ring footings for the case of $r_i/r_0 = 0.50$. The results allow to note that N'_{γ} increases considerably with increasing φ. The values of N'_{γ} for rough ring footings are clearly greater than those for smooth ring footings. The magnitude of N'_{γ} for a rough footing is found to be considerably larger than that of a smooth one for all values of φ. Fig. 3 illustrates the variation of N'_{γ} with r_i/r_0 for smooth and rough ring footings. It can be noted that the values of N'_{γ} decrease significantly with increasing r_i/r_0.

Table 1. Bearing capacity factor N'_{γ} for smooth and rough ring footings.

φ °	Smooth ring footings					Rough ring footings				
	Variation of r_i/r_0					Variation of r_i/r_0				
	0.00	0.25	0.33	0.50	0.75	0.00	0.25	0.33	0.50	0.75
20	1.37	0.86	0.75	0.55	0.30	2.71	1.75	1.50	1.09	0.60
25	3.20	2.06	1.76	1.26	0.66	6.78	4.61	3.83	2.68	1.42
30	7.71	5.14	4.31	2.98	1.50	17.5	12.9	10.5	6.93	3.43
35	19.7	13.7	11.5	7.60	3.65	44.5	37.1	29.8	18.3	8.37
40	55.5	40.2	33.1	21.6	9.78	133.7	132.4	113.5	65.9	26.7
45	179.5	136.6	112.6	72.1	30.8	460.9	446.2	430.8	274.4	97.5

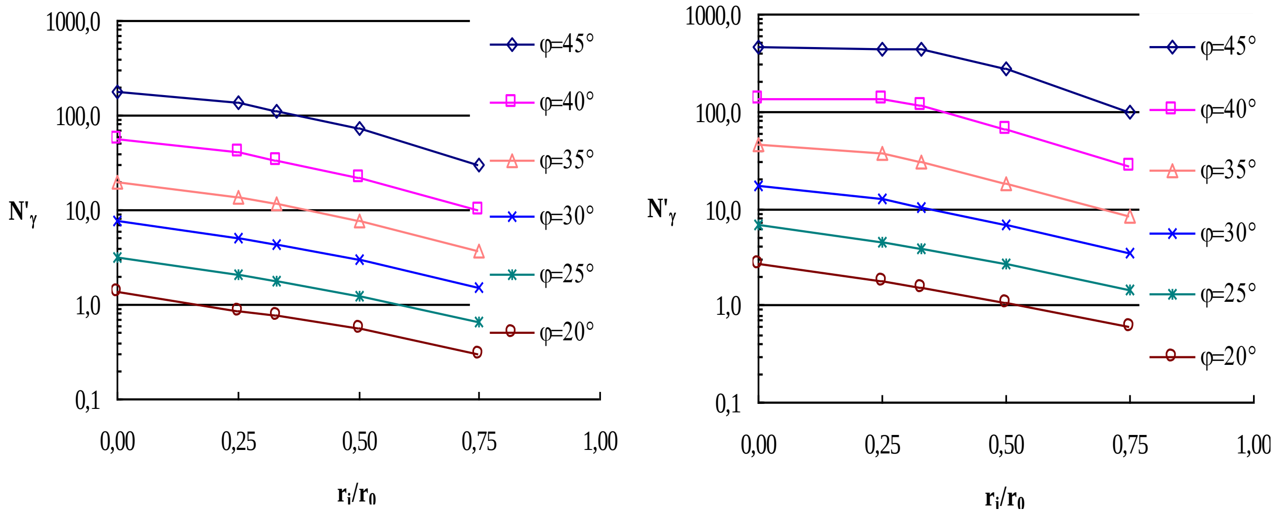
Fig. 4 shows the comparison, for a smooth ring footing, between the present results of N'_{γ} and the results of Choobbasti et al. (2010), Zhao and Wang (2008) and Kumar and Ghosh (2005) for two values of the friction soil φ = 25° and φ = 40°, and for a variable r_i/r_0. It can be noted that all the results of smooth N'_{γ} are close to each other, for both lower than for high angles of internal friction of the soil.

For a rough ring footing, the results given in Fig. 5 show a comparison of the obtained results of N'γ for rough footings with those of Kumar and Ghosh (2005), Choobbasti et al. (2010), Zhao and Wang (2008), and the experimental results of Saha (1978). The discussion of these results can be divided into two parts, first the comparison of the present study with the experimental results of Saha, then with the solutions of the other authors.



(a) **(b)**

Figure 2.Variation of the bearing capacity factor N'_γ with φ for a ring footing: (a) circular footing (r_i/r_0 = 0.0) and (b) ring footing (case of r_i/r_0 = 0.50).



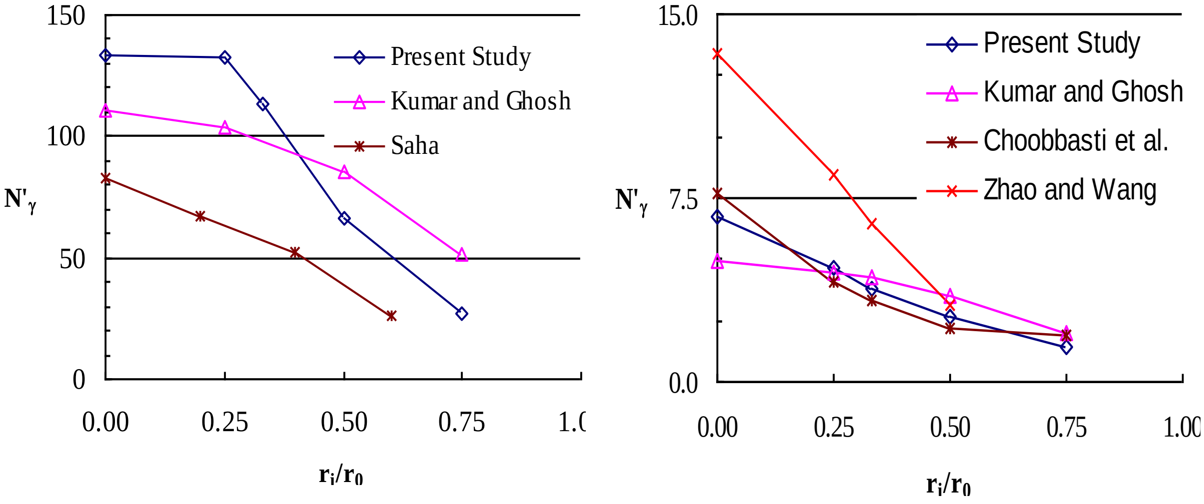
(a) **(b)**

Figure 3. Variation of the bearing capacity factor N'_γ for a ring footing with r_i/r_0: (a) smooth footing and (b) rough footing.

We can remark that for low angles of internal friction of the soil (φ<35°), the solutions of this study agree very well with the experimental results of Saha. However, for the high values of φ, the results of Saha are much lower than the present numerical solution. Also, as depicted by the figure 6, experimental results are lower than the Kumar solution too. This discrepancy between experimental and numerical solutions for high values of φ may be attributed to the actual behavior of soils which is nonassociated.

Regarding the comparison of this solution with the numerical results of the other authors, we can make the following remarks. For a low friction soil (φ<35°), we can note that the values of Zhao and Wang (2008) are the highest, particularly for r_i/r_0<0.50. On the other hand, the results of Choobbasti et al. (2010) and Kumar and Ghosh (2005) compare reasonably well with the present results for the range 0.25<r_i/r_0<0.75. For r_i/r_0=0, the results from the present analysis compare most favorably with the solution of Choobbasti et al. (2010). However, Kumar and Ghosh (2005) present a very low value of N'_γ which is less than the lower bound established by Lyamin et al. (2007).

For high friction angle (φ>35°), the results given in Fig. 5 show a comparison of the obtained results of N'_γ with those of Kumar and Ghosh (2005). The obtained results are larger for $r_i/r_0 \leq 0.33$ and smaller for $r_i/r_0 > 0.33$ compared to Kumar and Ghosh solutions. On the other hand, for $r_i/r_0=0$, the solution of Kumar and Ghosh is the lowest in the literature.



(a) **(b)**

Figure 4. Comparison of N'_γ values from this study with the results of other authors for rough footings and variable r_i/r_0: (a) φ=40° and (b) φ=25°.

5 CONCLUSIONS

The finite difference code FLAC was used to evaluate the soil bearing capacity factor N'_γ for smooth and rough ring footings for associated Mohr-Coulomb soils. From this investigation, comparing the obtained results presented in a form of design graphs and tables with the various available results given in the literature we can note the following:

- The magnitude of N'_γ is found to decrease continuously with an increase in r_i/r_0.
- Moreover, especially for greater values of φ, the magnitude of N'_γ for a rough footing is found to be significantly higher than the values for a smooth one. For φ=45°, the ratio reaches 257% and 317% for circular and ring footing with $r_i/r_0 = 0.75$ respectively.

In addition, the present values of N'_γ for smooth footings are found to match quite well with the various available results in the literature and they are all very close to each other. However, for rough footings, we note some discrepancy. The values provided by Bolton and Lau (1993) and Zhao and Wang (2008) are the highest and are more than 75% of the present results. Both present results and Kumar and Ghosh (2005) show differences more noticeable as φ increases and these differences vary with the ratio r_i/r_0. For $r_i/r_0<0.33$, the solution of Kumar and Ghosh is the lowest, but for $r_i/r_0> 0.33$, the present results are the lowest.

The comparison of the present numerical results with the experimental work of Saha (1978), shows a great agreement between the two solutions for low values of φ (φ<35°), however, for the high values of φ, the experimental results are much lower than the numerical ones. The discrepancy between experimental and numerical solutions for high values of φ may be attributed to the actual behavior of soils which is, in fact, nonassociated.

NOMENCLATURE

φ	angle of internal friction of the soil
ψ	dilation angle of the soil
γ	unit weight of the soil
φ^*	angle of internal friction of the equivalent associated soil
q_u	bearing capacity of the footing
r_i	internal radius of the ring footing
r_0	external radius of the ring footing
N'_γ	bearing capacity factor of the ring footing

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